CHAPTER 15

SET LANGUAGE

Detailed Mathematics Notes

1. Introduction to Sets

A set is a well-defined collection of distinct objects. The objects belonging to a set are called its elements or members.

$$A = \{1, 2, 3, 4, 5\}$$

Here A is a set and 1, 2, 3, 4, 5 are elements. We write $2 \in A$ and $7 \notin A$.

REMEMBER
A set must be well-defined, and repeated elements are written only once. $\{1,2,2,3,3\} = \{1,2,3\}$.

2. Representation of Sets

- Roster or Tabular Form: $A = \{2,4,6,8,10\}$.
- Set-Builder Form: $A = \{x : x \text{ is an even natural number less than } 12\}$.
- Descriptive Form: A is the set of even natural numbers less than 12.

3. Types of Sets

Empty Set: A set containing no element. Symbols: $\varnothing$ or $\{\}$. Example: $\{x \in N : x<0\}=\varnothing$.

Singleton Set: A set containing exactly one element, e.g. $\{7\}$.

Finite Set: A set containing a finite number of elements, e.g. $\{1,2,3,4,5\}$.

Infinite Set: A set containing infinitely many elements, e.g. $N=\{1,2,3,\ldots\}$.

4. Equal Sets

Two sets are equal if they contain exactly the same elements. For $A=\{1,2,3\}$ and $B=\{3,2,1\}$, $A=B$. Order does not matter.

5. Equivalent Sets

Two sets are equivalent if they contain the same number of elements. Equal sets have the same elements; equivalent sets only need the same cardinality.

$$n(A)=n(B)$$

6. Subset

If every element of A is also an element of B, then A is a subset of B.

$$A \subseteq B$$

Example: $\{1,2\} \subseteq \{1,2,3,4\}$.

7. Proper Subset

If $A \subseteq B$ but $A \neq B$, then A is a proper subset of B.

$$A \subset B$$

8. Important Facts About Subsets

- $A \subseteq A$ for every set A.
- $\varnothing \subseteq A$ for every set A.
- If a set contains n elements, total subsets = 2^n.
- Number of proper subsets = $2^n - 1$.

9. Power Set

The power set P(A) is the set of all subsets of A.

$$A=\{1,2\} \;\Rightarrow\; P(A)=\{\varnothing,\{1\},\{2\},\{1,2\}\}$$

$$n(P(A))=2^2=4$$

10. Universal Set

The universal set contains all objects under consideration and is usually represented by U.

$$U=\{1,2,3,4,5,6,7,8,9\}$$

11. Complement of a Set

The complement of A contains elements of U that do not belong to A.

$$A' = U - A$$

If $U=\{1,2,3,4,5,6\}$ and $A=\{2,4,6\}$, then $A'=\{1,3,5\}$.

12. Union of Sets

$A \cup B$ contains all elements belonging to A, B, or both.

$$A=\{1,2,3\},\ B=\{3,4,5\} \ \Rightarrow\ A\cup B=\{1,2,3,4,5\}$$

13. Intersection of Sets

$A \cap B$ contains elements common to both sets.

$$A\cap B=\{3\}$$

14. Disjoint Sets

Two sets are disjoint if they have no common element.

$$A\cap B=\varnothing$$

15. Difference of Sets

$A-B$ contains elements belonging to A but not B.

$$A-B=A\cap B'$$

Generally, $A-B \neq B-A$.

16. Symmetric Difference

The symmetric difference contains elements belonging to either set but not both.

$$A\triangle B=(A-B)\cup(B-A)$$

$$A=\{1,2,3\},\ B=\{3,4,5\} \ \Rightarrow\ A\triangle B=\{1,2,4,5\}$$

17. Cardinality of a Set

The number of elements in a set is its cardinality, represented by $n(A)$ or $|A|$.

$$A=\{2,4,6,8,10\} \ \Rightarrow\ n(A)=5$$

18. Cardinality of Union

$$n(A\cup B)=n(A)+n(B)-n(A\cap B)$$

The intersection is subtracted because common elements are counted twice.

Example: $n(A)=40$, $n(B)=30$, $n(A\cap B)=10 \Rightarrow n(A\cup B)=60$.

19. Three-Set Formula

$$n(A\cup B\cup C)=n(A)+n(B)+n(C)-n(A\cap B)-n(B\cap C)-n(C\cap A)+n(A\cap B\cap C)$$

The triple intersection is added back because it has been subtracted too many times.

20. Venn Diagram

A Venn diagram represents relationships between sets using circles. For two sets, the overlap represents $A\cap B$, the complete circles represent $A\cup B$, and the outside region represents the complement.

21. Two-Set Venn Diagram Example

If $n(U)=100$, $n(A)=60$, $n(B)=50$ and $n(A\cap B)=20$:

- Only A = $60-20 = 40$
- Only B = $50-20 = 30$
- Both = 20
- Neither = $100-(40+30+20) = 10$

22. Important Two-Set Relationships

- If A and B are disjoint, $n(A\cup B)=n(A)+n(B)$.
- If $A\subseteq B$, then $A\cap B=A$.
- If $A\subseteq B$, then $A\cup B=B$.

23. De Morgan's Laws

$$(A\cup B)'=A'\cap B'$$

$$(A\cap B)'=A'\cup B'$$

MEMORY RULE
Complement changes UNION into INTERSECTION and INTERSECTION into UNION.

24. Important Set Identities

Identity	Result
$A \cup \varnothing$	A
$A \cap U$	A
$A \cup U$	U
$A \cap \varnothing$	$\varnothing$
$A \cup A$	A
$A \cap A$	A
$A \cup A'$	U
$A \cap A'$	$\varnothing$
$(A')'$	A

25. Commutative Laws

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

26. Associative Laws

$$(A \cup B) \cup C = A \cup (B \cup C)$$

$$(A \cap B) \cap C = A \cap (B \cap C)$$

27. Distributive Laws

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

28. Difference and Complement

$$A - B = A \cap B'$$

$$B - A = B \cap A'$$

29. Number of Subsets

Total subsets = 2^n

Proper subsets = $2^n - 1$

$$\textbf{Non-empty subsets} = 2^n - 1$$

30. Number of Subsets with Exactly r Elements

$$\textbf{Number} = {}^nC_r$$

Example: A set has 5 elements. Subsets containing exactly 2 elements = ${}^5C_2 = 10$.

31. Even- and Odd-Sized Subsets

$$\textbf{Even-sized subsets} = 2^{n-1}$$

$$\textbf{Odd-sized subsets} = 2^{n-1}$$

These equalities hold for $n \geq 1$.

32. Cartesian Product

The Cartesian product $A \times B$ contains ordered pairs (a,b), where $a \in A$ and $b \in B$.

$$n(A \times B) = n(A)n(B)$$

If $A=\{1,2\}$ and $B=\{3,4,5\}$, then $A \times B$ contains 6 ordered pairs.

33. Set vs Element

If $A=\{1,2,3\}$, then $1 \in A$, whereas $\{1\} \subseteq A$. Membership and subset notation are not interchangeable.

34. Empty Set

$$\varnothing \subseteq A$$

The empty set is a subset of every set, but $\varnothing \in A$ is not necessarily true.

35. Solved Example 1

$A=\{1,2,3,4\}$, $B=\{3,4,5,6\}$. Find $A \cup B$ and $A \cap B$.

$$A \cup B = \{1,2,3,4,5,6\}$$

$$A \cap B = \{3,4\}$$

36. Solved Example 2

If $n(A)=45$, $n(B)=35$ and $n(A\cap B)=15$, find $n(A\cup B)$.

$$45+35-15=65$$

Answer: 65

37. Solved Example 3

In a group of 100 students, 60 study Mathematics, 45 study English, and 25 study both. How many study at least one subject?

$$n(M\cup E)=60+45-25=80$$

Answer: 80 students.

38. Solved Example 4

Using the previous data, how many students study neither Mathematics nor English?

$$100-80=20$$

Answer: 20 students.

39. Solved Example 5

If a set has 8 elements, find total, proper and non-empty subsets.

$$\text{Total} = 2^8 = 256$$

$$\text{Proper} = 256-1 = 255$$

$$\text{Non-empty} = 255$$

40. Solved Example 6

Let $A=\{1,2,3,4,5\}$. Find the number of subsets containing exactly 3 elements.

$$^5C_3 = 5!/(3!2!) = 10$$

Answer: 10.

41. Solved Example 7 — Three Sets

In a survey of 200 people, $n(A)=100$, $n(B)=90$, $n(C)=80$, $n(A\cap B)=40$, $n(B\cap C)=30$, $n(C\cap A)=35$ and $n(A\cap B\cap C)=15$. Find the number belonging to at least one set.

$$100+90+80-40-30-35+15=180$$

Answer: 180.

42. Solved Example 8 — De Morgan's Law

Simplify $(A\cup B)'$.

$$(A\cup B)'=A'\cap B'$$

43. Solved Example 9 — Set Difference

$A=\{2,4,6,8,10\}$, $B=\{4,8,12\}$. Find $A-B$.

$$A-B=\{2,6,10\}$$

44. Solved Example 10 — Cartesian Product

If $A=\{1,2,3\}$ and $B=\{a,b\}$, find $n(A\times B)$.

$$n(A\times B)=3\times 2=6$$

45. Common Question Patterns

- Identifying types of sets
- Subsets and proper subsets
- Power sets
- Number of subsets
- Union and intersection
- Difference and complement
- Disjoint sets
- Venn diagrams
- Two-set cardinality
- Three-set cardinality
- De Morgan's laws
- Set identities
- Counting students/persons
- At least one
- Neither
- Exactly one

- Exactly two
- Cartesian products
- Number of subsets with a fixed number of elements

46. QUICK REVISION

- Union $A \cup B$: all elements of A and B.
- Intersection $A \cap B$: common elements.
- Difference $A-B=A \cap B'$.
- Complement $A'=U-A$.
- Two-set cardinality: $n(A \cup B)=n(A)+n(B)-n(A \cap B)$.
- Total subsets of n-element set: 2^n.
- Proper and non-empty subsets: 2^n-1.
- Exactly r elements: ${}^nC_r$.
- Cartesian product: $n(A \times B)=n(A)n(B)$.
- De Morgan: complement changes union to intersection and intersection to union.

47. FORMULA & IDENTITY SHEET

Concept	Formula
Union	$A \cup B$
Intersection	$A \cap B$
Difference	$A-B=A \cap B'$
Complement	$A'=U-A$
Two-set cardinality	$n(A \cup B)=n(A)+n(B)-n(A \cap B)$
Three-set cardinality	$n(A \cup B \cup C)=\Sigma n(A)-\Sigma n(A \cap B)+n(A \cap B \cap C)$
Total subsets	2^n
Proper subsets	2^n-1
Non-empty subsets	2^n-1
Exactly r elements	${}^nC_r$
Even subsets	2^{n-1}
Odd subsets	2^{n-1}
Cartesian product	$n(A \times B)=n(A)n(B)$
De Morgan 1	$(A \cup B)'=A' \cap B'$
De Morgan 2	$(A \cap B)'=A' \cup B'$
Complement union	$A \cup A'=U$
Complement intersection	$A \cap A'=\varnothing$
Double complement	$(A')'=A$
Identity	$A \cup \varnothing=A$
Identity	$A \cap U=A$
Domination	$A \cup U=U$
Domination	$A \cap \varnothing=\varnothing$

FINAL TAKEAWAY

The most important areas are subsets, power sets, union/intersection, complements, Venn diagrams, cardinality formulas, De Morgan's laws and multi-set counting. Carefully distinguish 'at least one', 'exactly one', 'exactly two' and 'none'.