Chapter 4: Trigonometry

Trigonometric Ratios — Detailed Notes & Solved Examples

1. Introduction to Trigonometry

Trigonometry is the branch of mathematics that studies the relationship between the angles and sides of triangles.

Trigonometric ratios are defined using the sides of a right-angled triangle.

For an angle θ in a right-angled triangle, the three sides are identified as the perpendicular (opposite side), base (adjacent side), and hypotenuse.

2. The Six Trigonometric Ratios

Ratio	Definition	Reciprocal
sin θ	P/H	cosec θ
cos θ	B/H	sec θ
tan θ	P/B	cot θ
cosec θ	H/P	sin θ
sec θ	H/B	cos θ
cot θ	B/P	tan θ

3. SOH–CAH–TOA

A simple way to remember the three basic trigonometric ratios is SOH–CAH–TOA:

SOH → sin θ = Opposite / Hypotenuse

CAH → cos θ = Adjacent / Hypotenuse

TOA → tan θ = Opposite / Adjacent

Here, P denotes the perpendicular or opposite side, B denotes the base or adjacent side, and H denotes the hypotenuse.

4. Reciprocal Relationships

$$\csc\theta = 1/\sin\theta \qquad \sec\theta = 1/\cos\theta \qquad \cot\theta = 1/\tan\theta$$

$$\sin\theta \times \csc\theta = 1 \qquad \cos\theta \times \sec\theta = 1 \qquad \tan\theta \times \cot\theta = 1$$

5. Quotient Relationships

$$\tan\theta = \sin\theta / \cos\theta$$

$$\cot\theta = \cos\theta / \sin\theta$$

6. Pythagorean Relationships

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

For a right-angled triangle, the Pythagorean theorem is $H^2 = P^2 + B^2$.

7. Complementary Angles

Two angles are complementary when their sum is 90°. The following relationships are important:

$$\sin(90° - \theta) = \cos\theta$$

$$\cos(90° - \theta) = \sin\theta$$

$$\tan(90° - \theta) = \cot\theta$$

$$\cot(90° - \theta) = \tan\theta$$

$$\sec(90° - \theta) = \operatorname{cosec}\theta$$

$$\operatorname{cosec}(90° - \theta) = \sec\theta$$

8. Detailed Solved Examples

Example 1

In a right-angled triangle, the perpendicular is 3 cm and the base is 4 cm. Find all six trigonometric ratios of θ.

Given: P = 3 cm and B = 4 cm.

Using the Pythagorean theorem: $H^2 = P^2 + B^2 = 3^2 + 4^2 = 25$.

Therefore, H = 5 cm.

sin θ = P/H = 3/5.

cos θ = B/H = 4/5.

tan θ = P/B = 3/4.

cosec θ = H/P = 5/3.

sec θ = H/B = 5/4.

cot θ = B/P = 4/3.

Answer: sin θ = 3/5, cos θ = 4/5, tan θ = 3/4, cosec θ = 5/3, sec θ = 5/4, cot θ = 4/3.

Example 2

If sin θ = 5/13, where θ is an acute angle, find the remaining five trigonometric ratios.

Since sin θ = P/H, take P = 5 and H = 13.

Using $H^2 = P^2 + B^2$: $13^2 = 5^2 + B^2$.

$169 = 25 + B^2$, so $B^2 = 144$ and B = 12.

Therefore, cos θ = B/H = 12/13.

tan θ = P/B = 5/12.

cosec θ = H/P = 13/5.

sec θ = H/B = 13/12.

cot θ = B/P = 12/5.

Answer: cos θ = 12/13, tan θ = 5/12, cosec θ = 13/5, sec θ = 13/12, cot θ = 12/5.

Example 3

If cos θ = 8/17, find the remaining five trigonometric ratios.

Since cos θ = B/H, take B = 8 and H = 17.

Using $H^2 = P^2 + B^2$: $17^2 = P^2 + 8^2$.

$289 = P^2 + 64$, so $P^2 = 225$ and P = 15.

sin θ = P/H = 15/17.

tan θ = P/B = 15/8.

cosec θ = H/P = 17/15.

sec θ = H/B = 17/8.

cot θ = B/P = 8/15.

Answer: sin θ = 15/17, tan θ = 15/8, cosec θ = 17/15, sec θ = 17/8, cot θ = 8/15.

Example 4

If tan θ = 7/24, find sin θ and cos θ.

Since tan θ = P/B, take P = 7 and B = 24.

Using $H^2 = P^2 + B^2$: $H^2 = 7^2 + 24^2 = 625$.

Therefore, H = 25.

sin θ = P/H = 7/25.

cos θ = B/H = 24/25.

Answer: sin θ = 7/25 and cos θ = 24/25.

Example 5

A right-angled triangle has a hypotenuse of 10 cm and a perpendicular of 6 cm. Find sin θ, cos θ, and tan θ.

Given H = 10 cm and P = 6 cm.

Using $H^2 = P^2 + B^2$: $10^2 = 6^2 + B^2$.

$100 = 36 + B^2$, so $B^2 = 64$ and B = 8 cm.

sin θ = 6/10 = 3/5.

cos θ = 8/10 = 4/5.

tan θ = 6/8 = 3/4.

Answer: sin θ = 3/5, cos θ = 4/5, tan θ = 3/4.

Example 6

If sin θ = 4/7, find cosec θ.

Cosecant is the reciprocal of sine.

cosec θ = 1/sin θ = 1/(4/7).

Taking the reciprocal gives cosec θ = 7/4.

Answer: cosec θ = 7/4.

Example 7

If cos θ = 5/9, find sec θ.

Secant is the reciprocal of cosine.

sec θ = 1/cos θ = 1/(5/9).

Therefore, sec θ = 9/5.

Answer: sec θ = 9/5.

Example 8

If tan θ = 11/6, find cot θ.

Cotangent is the reciprocal of tangent.

cot θ = 1/tan θ = 1/(11/6).

Therefore, cot θ = 6/11.

Answer: cot θ = 6/11.

Example 9

If sin θ = 3/5 and cos θ = 4/5, find tan θ.

Use the quotient relationship: tan θ = sin θ / cos θ.

tan θ = (3/5)/(4/5).

tan θ = (3/5) × (5/4) = 3/4.

Answer: tan θ = 3/4.

Example 10

If sin θ = 5/13 and cos θ = 12/13, find cot θ.

Use cot θ = cos θ / sin θ.

cot θ = (12/13)/(5/13).

cot θ = (12/13) × (13/5) = 12/5.

Answer: cot θ = 12/5.

Example 11

The angle of a right-angled triangle is θ. If the hypotenuse is 20 cm and sin θ = 3/5, find the perpendicular.

sin θ = P/H.

Therefore, 3/5 = P/20.

Cross-multiplying: 5P = 60.

Hence, P = 12 cm.

Answer: P = 12 cm.

Example 12

In a right-angled triangle, cos θ = 4/5 and the hypotenuse is 25 cm. Find the base.

cos θ = B/H.

Therefore, 4/5 = B/25.

Cross-multiplying: 5B = 100.

Hence, B = 20 cm.

Answer: B = 20 cm.

Example 13

A right-angled triangle has perpendicular = 9 cm, base = 12 cm, and hypotenuse = 15 cm. Find tan θ.

By definition, tan θ = P/B.

Therefore, tan θ = 9/12.

Simplifying by dividing numerator and denominator by 3 gives tan θ = 3/4.

Answer: tan θ = 3/4.

Example 14

Which statement is always true for an acute angle θ? (A) sin θ > 1 (B) cos θ > 1 (C) The hypotenuse is greater than the perpendicular (D) The perpendicular is greater than the hypotenuse.

In every right-angled triangle, the hypotenuse is the longest side.

Therefore, H > P and H > B.

Hence, the correct option is C.

Answer: C. The hypotenuse is greater than the perpendicular.

Example 15

If tan θ = 3/4, find sin θ + cos θ.

Since tan θ = P/B = 3/4, take P = 3 and B = 4.

The hypotenuse is $H = \sqrt{(3^2 + 4^2)} = 5$.

Therefore, sin θ = 3/5 and cos θ = 4/5.

Thus, sin θ + cos θ = 3/5 + 4/5 = 7/5.

Answer: 7/5.

9. Important Problem-Solving Method

Whenever a question gives one trigonometric ratio, use the following systematic method:

1. Write the definition of the given ratio.
2. Identify the two corresponding sides.
3. Assign convenient values to those sides according to the given ratio.
4. Use the Pythagorean theorem to find the third side.
5. Calculate the required trigonometric ratio.

10. Common Pythagorean Triplets

Perpendicular	Base	Hypotenuse
3	4	5
5	12	13
7	24	25
8	15	17
9	40	41
12	35	37
20	21	29

11. Quick Revision Formula Sheet

sin θ = P/H   cos θ = B/H   tan θ = P/B

cosec θ = H/P   sec θ = H/B   cot θ = B/P

$$\tan\theta = \sin\theta/\cos\theta \qquad \cot\theta = \cos\theta/\sin\theta$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \mathrm{cosec}^2\theta$$

$$H^2 = P^2 + B^2$$

12. Key Points to Remember

- The hypotenuse is always the side opposite the right angle and is the longest side.
- The terms perpendicular and base depend on the angle being considered.
- Use SOH–CAH–TOA to remember the three basic ratios.
- Sine and cosecant are reciprocals; cosine and secant are reciprocals; tangent and cotangent are reciprocals.
- For an acute angle in a right-angled triangle, all six trigonometric ratios are positive.
- A triangle-based approach is one of the most reliable ways to solve ratio questions.