Chapter 4: Trigonometry

Standard Angles — Detailed Notes & Solved Examples

1. Meaning of Standard Angles

Certain angles occur repeatedly in trigonometric calculations. These are called standard angles.

0°, 30°, 45°, 60°, 90°

The values of the six trigonometric ratios for these angles can be calculated exactly.

2. Standard-Angle Values

θ	sin θ	cos θ	tan θ	cosec θ	sec θ	cot θ
0°	0	1	0	Not defined	1	Not defined
30°	1/2	√3/2	1/√3	2	2/√3	√3
45°	1/√2	1/√2	1	√2	√2	1
60°	√3/2	1/2	√3	2/√3	2	1/√3
90°	1	0	Not defined	1	Not defined	0

3. Easy Method to Remember Sine Values

For 0°, 30°, 45°, 60° and 90°, the sine values can be remembered as:

$$\sqrt{0}/2,\ \sqrt{1}/2,\ \sqrt{2}/2,\ \sqrt{3}/2,\ \sqrt{4}/2$$

Therefore:

$$\sin 0° = 0 \quad \sin 30° = 1/2 \quad \sin 45° = 1/\sqrt{2}$$

$$\sin 60° = \sqrt{3}/2 \quad \sin 90° = 1$$

4. Easy Method to Remember Cosine Values

The cosine values are the sine values written in reverse order:

$$1,\ \sqrt{3}/2,\ 1/\sqrt{2},\ 1/2,\ 0$$

This also follows from the complementary-angle relationship:

$$\cos\theta = \sin(90° - \theta)$$

5. Finding Tangent Values

Use the quotient identity:

$$\tan\theta = \sin\theta / \cos\theta$$

$$\tan 30° = 1/\sqrt{3} \quad \tan 45° = 1 \quad \tan 60° = \sqrt{3}$$

At 90°, cosine is zero, so tan 90° is not defined.

6. Derivation of 45° Values

Consider an isosceles right-angled triangle with the two perpendicular sides equal to 1 unit.

$$H^2 = 1^2 + 1^2 = 2 \Rightarrow H = \sqrt{2}$$

Each acute angle is 45°.

$$\sin 45° = 1/\sqrt{2}$$

$$\cos 45° = 1/\sqrt{2}$$

$$\tan 45° = 1$$

7. Derivation of 30° and 60° Values

Consider an equilateral triangle with each side equal to 2 units. Draw a perpendicular from one vertex to the opposite side. The triangle is divided into two identical right-angled triangles.

For each right-angled triangle, the hypotenuse is 2 units, the base is 1 unit, and the perpendicular is √3 units.

$$\sin 30° = 1/2 \quad \cos 30° = \sqrt{3}/2 \quad \tan 30° = 1/\sqrt{3}$$

$$\sin 60° = \sqrt{3}/2 \quad \cos 60° = 1/2 \quad \tan 60° = \sqrt{3}$$

8. Reciprocal Values

$$\operatorname{cosec} \theta = 1/\sin \theta \quad \sec \theta = 1/\cos \theta \quad \cot \theta = 1/\tan \theta$$

Examples:

$$\operatorname{cosec} 30° = 2 \quad \sec 60° = 2 \quad \cot 45° = 1$$

9. Important Relationships

$$\sin 30° = \cos 60° = 1/2$$

$$\sin 60° = \cos 30° = \sqrt{3}/2$$

$$\sin 45° = \cos 45° = 1/\sqrt{2}$$

$$\tan 30° = \cot 60° = 1/\sqrt{3}$$

$$\tan 60° = \cot 30° = \sqrt{3}$$

$$\tan 45° = \cot 45° = 1$$

10. Detailed Solved Examples

Example 1

Find sin 30°.

From the standard-angle table, the value of sine at 30° is 1/2.

Answer: sin 30° = 1/2

Example 2

Find cos 60°.

From the standard-angle table, the value of cosine at 60° is 1/2.

Answer: cos 60° = 1/2

Example 3

Find tan 45°.

Use tan θ = sin θ/cos θ.

sin 45° = 1/√2 and cos 45° = 1/√2.

Therefore tan 45° = (1/√2)/(1/√2) = 1.

Answer: tan 45° = 1

Example 4

Find sin 60° + cos 60°.

sin 60° = √3/2 and cos 60° = 1/2.

Therefore √3/2 + 1/2 = (√3 + 1)/2.

Answer: (√3 + 1)/2

Example 5

Find tan 60° – tan 30°.

tan 60° = √3 and tan 30° = 1/√3.

Therefore √3 – 1/√3 = (3 – 1)/√3 = 2/√3.

Rationalising gives 2√3/3.

Answer: 2√3/3

Example 6

Find sec 60°.

sec θ = 1/cos θ.

cos 60° = 1/2.

Therefore sec 60° = 1/(1/2) = 2.

Answer: 2

Example 7

Find cosec 30°.

cosec θ = 1/sin θ.

sin 30° = 1/2.

Therefore cosec 30° = 2.

Answer: 2

Example 8

Find cot 60°.

$\cot \theta = 1/\tan \theta$.

$\tan 60° = \sqrt{3}$.

Therefore $\cot 60° = 1/\sqrt{3} = \sqrt{3}/3$ after rationalisation.

Answer: $\sqrt{3}/3$

Example 9

Simplify 2 sin 30° + 3 cos 60°.

Substitute $\sin 30° = 1/2$ and $\cos 60° = 1/2$.

$2(1/2) + 3(1/2) = 1 + 3/2$.

Therefore the result is 5/2.

Answer: 5/2

Example 10

Simplify sin 30° cos 60°.

Substitute the standard values: $(1/2)(1/2)$.

Multiplying gives 1/4.

Answer: 1/4

Example 11

Simplify sin 60°/cos 60°.

Substitute $\sin 60° = \sqrt{3}/2$ and $\cos 60° = 1/2$.

$(\sqrt{3}/2)/(1/2) = \sqrt{3}$.

This is also tan 60°.

Answer: $\sqrt{3}$

Example 12

Simplify cos 30°/sin 30°.

Substitute $\cos 30° = \sqrt{3}/2$ and $\sin 30° = 1/2$.

$(\sqrt{3}/2)/(1/2) = \sqrt{3}$.

This is also cot 30°.

Answer: $\sqrt{3}$

Example 13

Find $\sin^2 30° + \cos^2 30°$.

sin 30° = 1/2 and cos 30° = √3/2.

Therefore $(1/2)^2 + (\sqrt{3}/2)^2 = 1/4 + 3/4 = 1$.

Answer: 1

Example 14

Find tan 30° × tan 60°.

tan 30° = 1/√3 and tan 60° = √3.

Their product is (1/√3)(√3) = 1.

Answer: 1

Example 15

Find sec 60° + cosec 30°.

sec 60° = 2 and cosec 30° = 2.

Therefore 2 + 2 = 4.

Answer: 4

11. Special Observations

- sin 30° = cos 60° = 1/2.
- sin 60° = cos 30° = √3/2.
- sin 45° = cos 45°.
- tan 45° = 1.
- tan 30° × tan 60° = 1.

12. Common Mistakes

- Do not confuse sin 30° with cos 30°. They are 1/2 and √3/2 respectively.
- Do not confuse tan 30° with tan 60°. They are 1/√3 and √3 respectively.
- tan 90° is not zero; it is not defined because cos 90° = 0.
- Remember that secant, cosecant, and cotangent are reciprocals of cosine, sine, and tangent respectively.

13. Quick Revision Table

Angle	sin θ	cos θ	tan θ
0°	0	1	0
30°	1/2	√3/2	1/√3
45°	1/√2	1/√2	1
60°	√3/2	1/2	√3
90°	1	0	Not defined

14. Final Formula Sheet

sin 0° = 0 sin 30° = 1/2 sin 45° = 1/√2 sin 60° = √3/2 sin 90° = 1

cos 0° = 1 cos 30° = √3/2 cos 45° = 1/√2 cos 60° = 1/2 cos 90° = 0

$\tan 0° = 0$ $\tan 30° = 1/\sqrt{3}$ $\tan 45° = 1$ $\tan 60° = \sqrt{3}$

$\tan 90°$ = Not Defined

Most important relationships:

$\sin 30° = \cos 60° = 1/2$

$\sin 60° = \cos 30° = \sqrt{3}/2$

$\sin 45° = \cos 45° = 1/\sqrt{2}$

$\tan 30° = 1/\sqrt{3}$ $\tan 45° = 1$ $\tan 60° = \sqrt{3}$