Chapter 4: Trigonometry

Heights & Distances — Detailed Notes & Solved Examples

1. Introduction

Heights and distances is an important application of trigonometry. It uses trigonometric ratios to calculate the height of an object or the distance between an observer and an object.

Typical objects include buildings, towers, trees, poles, cliffs, and aircraft. The questions are generally represented by a right-angled triangle.

2. Basic Terms

Height: The vertical distance from the base of an object to its top.

Distance: The horizontal distance between the observer and the foot of the object.

Line of sight: The straight line joining the observer's eye to the point being observed.

Angle of elevation: The angle between the horizontal line through the observer's eye and the line of sight when the object is above the observer.

Angle of depression: The angle between the horizontal line through the observer's eye and the line of sight when the object is below the observer.

3. Angle of Elevation

Suppose an observer stands at point A and looks upward at the top B of a vertical object. If AC is horizontal and AB is the line of sight, then the angle BAC is the angle of elevation.

tan θ = Height / Horizontal Distance

Therefore, if the horizontal distance and angle of elevation are known:

Height = Distance × tan θ

If the height and angle are known:

Distance = Height / tan θ

4. Angle of Depression

When an observer looks downward from a higher point toward an object below, the angle formed between the horizontal line and the line of sight is called the angle of depression.

If the horizontal lines are parallel, the angle of depression is equal to the corresponding angle of elevation at the object.

Angle of depression = Corresponding angle of elevation

5. Choosing the Correct Trigonometric Ratio

The first step is to identify the three sides of the right-angled triangle.

sin θ = Perpendicular / Hypotenuse

cos θ = Base / Hypotenuse

tan θ = Perpendicular / Base

In most basic heights-and-distances problems, tangent is the most convenient ratio because height and horizontal distance form the two perpendicular sides.

6. Standard Angles Used

The standard angles 30°, 45°, and 60° are frequently used.

$$\tan 30^\circ = 1/\sqrt{3}$$

$$\tan 45^\circ = 1$$

$$\tan 60^\circ = \sqrt{3}$$

7. General Problem-Solving Method

1. Read the question carefully and identify the object, observer, height, and distance.
2. Draw a simple diagram whenever possible.
3. Mark the given angle and identify the right angle.
4. Identify the opposite, adjacent, and hypotenuse sides with respect to the given angle.
5. Select the appropriate trigonometric ratio.
6. Substitute the known values.
7. Solve the resulting equation and include the correct unit.

8. Detailed Solved Examples

Example 1

A person stands 20 m away from the foot of a tower. The angle of elevation of the top of the tower is 45°. Find the height of the tower.

Let the height of the tower be h metres.

The horizontal distance is 20 m and the angle of elevation is 45°.

Using tan θ = Height/Distance:

$\tan 45^\circ = h/20$.

Since $\tan 45^\circ = 1$, we get $1 = h/20$.

Therefore, $h = 20$ m.

Answer: 20 m

Example 2

A pole casts a shadow 10 m long when the angle of elevation of the sun is 60°. Find the height of the pole.

Let the height of the pole be h metres.

The length of the shadow is 10 m.

Using tan θ = Height/Shadow:

$\tan 60^\circ = h/10$.

Since tan 60° = √3, h/10 = √3.

Therefore, h = 10√3 m.

Answer: 10√3 m

Example 3

From a point 30 m from the base of a tower, the angle of elevation of its top is 30°. Find the height of the tower.

Let the height be h metres.

Horizontal distance = 30 m.

tan 30° = h/30.

Since tan 30° = 1/√3, h/30 = 1/√3.

Therefore, h = 30/√3 = 10√3 m.

Answer: 10√3 m

Example 4

The angle of elevation of the top of a building from a point on the ground is 60°. If the point is 15 m from the building, find the height of the building.

Let the height be h metres.

Distance from the building = 15 m.

tan 60° = h/15.

√3 = h/15.

Therefore, h = 15√3 m.

Answer: 15√3 m

Example 5

A ladder 10 m long rests against a vertical wall and makes an angle of 60° with the ground. Find the height reached by the ladder on the wall.

The ladder is the hypotenuse, so H = 10 m.

The required height is the side opposite 60°.

Use sin 60° = Height/Hypotenuse.

√3/2 = h/10.

Therefore, h = 5√3 m.

Answer: 5√3 m

Example 6

A ladder 10 m long makes an angle of 60° with the ground. Find the distance of the foot of the ladder from the wall.

The required horizontal distance is adjacent to 60°.

Use cos 60° = Base/Hypotenuse.

1/2 = d/10.

Therefore, d = 5 m.

Answer: 5 m

Example 7

From the top of a 20 m high building, the angle of depression of a car on the ground is 45°. Find the horizontal distance of the car from the building.

The angle of depression is equal to the corresponding angle of elevation, so the angle at the car is 45°.

Let the horizontal distance be d metres.

$\tan 45° = 20/d$.

Since $\tan 45° = 1$, $1 = 20/d$.

Therefore, d = 20 m.

Answer: 20 m

Example 8

From the top of a 30 m high tower, the angle of depression of a point on the ground is 30°. Find the distance of the point from the foot of the tower.

The corresponding angle of elevation is 30°.

Let the horizontal distance be d metres.

$\tan 30° = 30/d$.

$1/\sqrt{3} = 30/d$.

Therefore, $d = 30\sqrt{3}$ m.

Answer: $30\sqrt{3}$ m

Example 9

A tree is 12 m high. If the angle of elevation of its top from a point on the ground is 30°, find the distance of the point from the tree.

Let the horizontal distance be d metres.

$\tan 30° = 12/d$.

$1/\sqrt{3} = 12/d$.

Therefore, $d = 12\sqrt{3}$ m.

Answer: $12\sqrt{3}$ m

Example 10

A man observes the top of a tower at an angle of elevation of 45°. If the height of the tower is 25 m, find the distance of the man from the tower.

Let the distance be d metres.

$\tan 45° = 25/d$.

Since $\tan 45° = 1$, $1 = 25/d$.

Therefore, d = 25 m.

Answer: 25 m

Example 11

A tower is observed from a point 40 m away. The angle of elevation of the top is 45°. If the observer's eye is at ground level, find the height of the tower.

Let the height be h metres.

$\tan 45° = h/40$.

Since $\tan 45° = 1$, $h/40 = 1$.

Therefore, $h = 40$ m.

Answer: 40 m

Example 12

A vertical pole is 8 m high. From a point on the ground, the angle of elevation of its top is 60°. Find the distance between the point and the foot of the pole.

Let the distance be d metres.

$\tan 60° = 8/d$.

$\sqrt{3} = 8/d$.

Therefore, $d = 8/\sqrt{3} = 8\sqrt{3}/3$ m.

Answer: $8\sqrt{3}/3$ m

Example 13

Two points are on the same straight line with the foot of a tower. From the nearer point, the angle of elevation of the top is 60°, and from the farther point it is 30°. If the distance between the two points is 20 m, find the height of the tower.

Let the distance from the foot of the tower to the nearer point be x metres. Then the distance to the farther point is x + 20 metres.

Using the nearer point: $\tan 60° = h/x$, so $h = x\sqrt{3}$.

Using the farther point: $\tan 30° = h/(x + 20)$, so $h = (x + 20)/\sqrt{3}$.

Equate the two expressions: $x\sqrt{3} = (x + 20)/\sqrt{3}$.

Multiplying by $\sqrt{3}$: $3x = x + 20$.

Therefore, $2x = 20$ and $x = 10$ m.

Hence, $h = 10\sqrt{3}$ m.

Answer: $10\sqrt{3}$ m

Example 14

From the top of a tower, the angle of depression of a point on the ground is 60°. If the horizontal distance is 20 m, find the height of the tower.

The angle of depression equals the corresponding angle of elevation, so the angle is 60°.

Let the tower height be h metres.

tan 60° = h/20.

√3 = h/20.

Therefore, h = 20√3 m.

Answer: 20√3 m

Example 15

A person standing 50 m from a building observes its top at an angle of elevation of 30°. If the person's eye level is 1.5 m above the ground, find the height of the building.

Let the height above eye level be h metres.

tan 30° = h/50.

Therefore, h = 50/√3 = 50√3/3 m.

The total height of the building is h + 1.5.

Therefore, total height = 50√3/3 + 1.5 m.

Answer: 50√3/3 + 1.5 m

Example 16

A kite is flying at a height of 30 m. If the string makes an angle of 60° with the horizontal ground, find the length of the string, assuming the string is straight.

The height is the side opposite 60° and the string is the hypotenuse.

Use sin 60° = Height/String.

√3/2 = 30/L.

Therefore, L = 60/√3 = 20√3 m.

Answer: 20√3 m

9. Angle of Elevation and Depression — Important Rule

When two horizontal lines are parallel, the angle of depression from the upper point is equal to the corresponding angle of elevation from the lower point.

Angle of depression = Angle of elevation

This rule is especially useful in questions involving towers, buildings, cliffs, aircraft, and ships.

10. Heights and Distances Formula Sheet

tan θ = Height / Horizontal Distance

Height = Horizontal Distance × tan θ

Horizontal Distance = Height / tan θ

sin θ = Height / Line of Sight

Height = Line of Sight × sin θ

$$\cos\theta = \text{Horizontal Distance} / \text{Line of Sight}$$

$$\text{Horizontal Distance} = \text{Line of Sight} \times \cos\theta$$

11. Important Standard Values

$$\tan 30° = 1/\sqrt{3} = \sqrt{3}/3$$

$$\tan 45° = 1$$

$$\tan 60° = \sqrt{3}$$

$$\sin 30° = 1/2$$

$$\sin 60° = \sqrt{3}/2$$

$$\cos 30° = \sqrt{3}/2$$

$$\cos 60° = 1/2$$

12. Common Mistakes

- Do not confuse the angle of elevation with the angle of depression.
- Always identify the angle with respect to the horizontal line.
- Do not automatically use tangent; first identify the known and required sides.
- Remember that the line of sight is usually the hypotenuse.
- Include units such as metres or centimetres in the final answer.
- When the observer's eye is above ground level, add the eye height to the calculated height above eye level.
- Draw a diagram for complicated problems, especially when two observation points are involved.

13. Quick Revision

Concept	Key Relationship
Angle of elevation	Object is above the observer's horizontal line.
Angle of depression	Object is below the observer's horizontal line.
Tangent	$\tan\theta$ = Height / Horizontal Distance
Sine	$\sin\theta$ = Height / Line of Sight
Cosine	$\cos\theta$ = Horizontal Distance / Line of Sight
Depression-elevation rule	Angle of depression = Corresponding angle of elevation

14. Final Problem-Solving Checklist

✓ Draw the right-angled triangle.

✓ Mark the known angle.

✓ Identify the height, horizontal distance, and line of sight.

✓ Choose sin, cos, or tan according to the sides involved.

✓ Substitute the standard-angle value if required.

✓ Solve carefully and write the final answer with its unit.