Chapter 5: Plane Geometry & Loci

Chapter 3 — Triangles | Detailed Notes & Solved Examples

1. What Is a Triangle?

A triangle is a closed plane figure formed by three line segments. It has three sides, three vertices and three interior angles.

Sum of the three interior angles of a triangle = 180°

Triangles are one of the basic building blocks of geometry. Their properties are used extensively in problems involving angles, lengths, areas and constructions.

2. Parts of a Triangle

The three line segments are called its sides, the three meeting points are its vertices, and the three angles formed inside the triangle are its interior angles.

In triangle ABC, the sides are AB, BC and CA, while the angles are ∠A, ∠B and ∠C.

3. Classification of Triangles by Sides

Type	Side Property	Key Point
Equilateral	All three sides are equal	All three angles are 60°
Isosceles	Two sides are equal	Angles opposite equal sides are equal
Scalene	All three sides are different	No two sides or angles need be equal

4. Classification of Triangles by Angles

Type	Angle Property	Key Point
Acute-angled	All three angles are less than 90°	No right angle
Right-angled	One angle is exactly 90°	The side opposite 90° is the hypotenuse
Obtuse-angled	One angle is greater than 90°	The other two angles are acute

5. Angle Sum Property

The three interior angles of every triangle add up to 180°.

$$\angle A + \angle B + \angle C = 180°$$

This property works for every type of triangle—equilateral, isosceles, scalene, acute, right or obtuse.

6. Exterior Angle of a Triangle

When one side of a triangle is extended, the angle formed outside the triangle is called an exterior angle.

An exterior angle is equal to the sum of the two interior opposite angles.

Exterior angle = Sum of the two opposite interior angles

The exterior angle and its adjacent interior angle form a linear pair, so their sum is 180°.

7. Important Exterior-Angle Results

Exterior angle = Opposite interior angle 1 + Opposite interior angle 2

Exterior angle + Adjacent interior angle = 180°

8. Isosceles Triangle

An isosceles triangle has two equal sides. The angles opposite those equal sides are also equal.

Equal sides → Equal opposite angles

Conversely, if two angles of a triangle are equal, the sides opposite those angles are equal.

9. Equilateral Triangle

An equilateral triangle has three equal sides. Because equal sides have equal opposite angles and the angle sum is 180°, each angle is 60°.

Each angle of an equilateral triangle = 60°

10. Right-Angled Triangle

A right-angled triangle has one angle equal to 90°. The side opposite the right angle is called the hypotenuse and is the longest side.

$\text{Hypotenuse}^2 = \text{Perpendicular}^2 + \text{Base}^2$

This is the Pythagorean theorem.

11. Pythagorean Theorem

In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$H^2 = P^2 + B^2$

The theorem can be used to find an unknown side when the other two sides are known.

12. Converse of Pythagorean Theorem

If, in a triangle, the square of the longest side is equal to the sum of the squares of the other two sides, then the triangle is right-angled.

If $c^2 = a^2 + b^2$, then the angle opposite c is 90°.

13. Triangle Inequality

The sum of any two sides of a triangle must be greater than the third side.

$a + b > c$

$b + c > a$

$c + a > b$

This condition can be used to determine whether three given lengths can form a triangle.

14. Relation Between Sides and Angles

In any triangle, the larger side is opposite the larger angle, and the smaller side is opposite the smaller angle.

Therefore, if one side is longer than another, the angle opposite the longer side is also larger.

15. Congruence of Triangles

Two triangles are congruent when they have exactly the same shape and size. Their corresponding sides and corresponding angles are equal.

The important congruence criteria are:

SSS — Side, Side, Side

SAS — Side, Angle, Side

ASA — Angle, Side, Angle

AAS — Angle, Angle, Side

RHS — Right angle, Hypotenuse, Side

16. SSS Congruence

If the three sides of one triangle are equal respectively to the three sides of another triangle, the two triangles are congruent.

17. SAS Congruence

If two sides and the included angle of one triangle are equal respectively to two sides and the included angle of another triangle, the triangles are congruent.

18. ASA Congruence

If two angles and the included side of one triangle are equal respectively to two angles and the included side of another triangle, the triangles are congruent.

19. AAS Congruence

If two angles and a corresponding non-included side are equal in two triangles, the triangles are congruent.

20. RHS Congruence

For two right-angled triangles, if the hypotenuse and one corresponding side are equal, the triangles are congruent.

21. Similarity of Triangles

Similar triangles have the same shape, although their sizes may be different. Corresponding angles are equal and corresponding sides are proportional.

The common similarity criteria are:

AA — Angle, Angle

SSS — Proportional corresponding sides

SAS — Proportional two sides with equal included angle

22. Properties of Similar Triangles

Corresponding angles are equal.

Corresponding sides are proportional.

If the ratio of corresponding sides is k, then the ratio of their areas is k^2.

23. Area of a Triangle

The area of a triangle is half the product of its base and the corresponding height.

$$\text{Area} = 1/2 \times \text{Base} \times \text{Height}$$

The height must be measured perpendicular to the chosen base.

24. Median of a Triangle

A median is a line segment joining a vertex of a triangle to the midpoint of the opposite side.

Every triangle has three medians, and all three medians meet at a common point called the centroid.

Centroid divides each median in the ratio 2 : 1

25. Altitude of a Triangle

An altitude is a perpendicular line segment drawn from a vertex to the opposite side or its extension.

A triangle has three altitudes. They meet at a point called the orthocentre.

26. Angle Bisector

An angle bisector divides an angle into two equal angles.

The three internal angle bisectors of a triangle meet at the incentre.

27. Perpendicular Bisector

The perpendicular bisector of a side passes through its midpoint and is perpendicular to the side.

The perpendicular bisectors of the three sides meet at the circumcentre, which is equidistant from all three vertices.

28. Important Centres of a Triangle

Centre	Formed by	Important Property
Centroid	Three medians	Divides each median in 2:1
Incentre	Three angle bisectors	Equidistant from all three sides
Circumcentre	Three perpendicular bisectors	Equidistant from all three vertices
Orthocentre	Three altitudes	Point where all three altitudes meet

29. Detailed Solved Examples

Example 1

The angles of a triangle are 50°, 60° and x°. Find x.

The angles of a triangle add up to 180°.

50° + 60° + x = 180°.

x = 70°.

Answer: 70°

Example 2

Two angles of a triangle are 45° and 65°. Find the third angle.

Let the third angle be x.

45° + 65° + x = 180°.

110° + x = 180°.

Therefore x = 70°.

Answer: 70°

Example 3

An exterior angle of a triangle is 120°. One of its opposite interior angles is 50°. Find the other opposite interior angle.

The exterior angle equals the sum of the two opposite interior angles.

120° = 50° + x.

Therefore x = 70°.

Answer: 70°

Example 4

The two equal angles of an isosceles triangle are 55° each. Find the third angle.

The sum of the angles of a triangle is 180°.

55° + 55° + x = 180°.

110° + x = 180°.

Therefore x = 70°.

Answer: 70°

Example 5

The angles of a triangle are in the ratio 2:3:4. Find the angles.

Let the angles be 2x°, 3x° and 4x°.

Their sum is 180°: 2x + 3x + 4x = 180°.

9x = 180°.

x = 20°.

Therefore, the angles are 40°, 60° and 80°.

Answer: 40°, 60° and 80°

Example 6

The sides of a triangle are 5 cm, 12 cm and 13 cm. Determine whether the triangle is right-angled.

The longest side is 13 cm.

Check the Pythagorean relationship: $13^2 = 5^2 + 12^2$.

$169 = 25 + 144 = 169$.

The condition is satisfied.

Therefore the triangle is right-angled.

Answer: Yes, it is a right-angled triangle.

Example 7

A right-angled triangle has perpendicular 6 cm and base 8 cm. Find its hypotenuse.

Use $H^2 = P^2 + B^2$.

$H^2 = 6^2 + 8^2 = 36 + 64 = 100$.

Therefore H = 10 cm.

Answer: 10 cm

Example 8

A right-angled triangle has hypotenuse 13 cm and one side 5 cm. Find the other side.

Let the unknown side be x cm.

$13^2 = 5^2 + x^2$.

$169 = 25 + x^2$.

$x^2 = 144$.

Therefore x = 12 cm.

Answer: 12 cm

Example 9

Can 4 cm, 7 cm and 12 cm form a triangle?

The sum of any two sides must be greater than the third side.

Check the two smaller sides: 4 + 7 = 11.

Since 11 is not greater than 12, the triangle inequality is violated.

Answer: No, these lengths cannot form a triangle.

Example 10

Two triangles have sides 5 cm, 7 cm and 9 cm respectively. The corresponding sides of another triangle are also 5 cm, 7 cm and 9 cm. Are the triangles congruent?

All three corresponding sides are equal.

Therefore the SSS congruence criterion applies.

Answer: Yes, the triangles are congruent by SSS.

Example 11

In two triangles, two sides are equal respectively and the included angle between them is also equal. Which congruence criterion applies?

Two corresponding sides are equal.

The included angle is equal.

Therefore the SAS criterion applies.

Answer: SAS congruence

Example 12

Two similar triangles have corresponding sides in the ratio 2:3. Find the ratio of their areas.

For similar triangles, the ratio of areas is the square of the ratio of corresponding sides.

Area ratio = $(2/3)^2$.

Therefore the ratio is 4:9.

Answer: 4:9

Example 13

The base of a triangle is 12 cm and its perpendicular height is 7 cm. Find its area.

Area = 1/2 × base × height.

Area = 1/2 × 12 × 7.

Area = 42 cm^2.

Answer: 42 cm^2

Example 14

A triangle has sides 7 cm, 8 cm and 9 cm. Which angle is the largest?

The largest angle is opposite the longest side.

The longest side is 9 cm.

Therefore, the angle opposite the 9 cm side is the largest.

Answer: The angle opposite the 9 cm side.

Example 15

An equilateral triangle has a side of 10 cm. Find each angle.

All sides of an equilateral triangle are equal.

Therefore all three angles are equal.

The angle sum is 180°.

Each angle = 180°/3 = 60°.

Answer: 60°

Example 16

A median divides the opposite side of a triangle into what ratio?

A median joins a vertex to the midpoint of the opposite side.

Therefore it divides that side into two equal parts.

Answer: 1:1

Example 17

The centroid divides a median in what ratio?

The centroid lies on each median.

It divides each median so that the segment from the vertex to the centroid is twice the segment from the centroid to the midpoint.

Answer: 2:1

Example 18

The three angle bisectors of a triangle meet at which point?

The internal angle bisectors of a triangle are concurrent.

Their common point is called the incentre.

Answer: Incentre

30. Important Triangle Theorems

- The sum of the interior angles of a triangle is 180°.
- An exterior angle is equal to the sum of the two opposite interior angles.
- Angles opposite equal sides are equal.
- Sides opposite equal angles are equal.
- The hypotenuse is the longest side of a right-angled triangle.
- Pythagorean theorem: $H^2 = P^2 + B^2$.
- The sum of any two sides of a triangle is greater than the third side.
- In similar triangles, corresponding angles are equal and corresponding sides are proportional.

31. Common Mistakes

- Do not use the Pythagorean theorem unless the triangle is right-angled.
- Remember that the exterior angle is formed by extending one side of the triangle.
- In an isosceles triangle, equal sides have equal opposite angles.
- Do not confuse congruent triangles with similar triangles: congruent triangles have the same size and shape, while similar triangles have the same shape but may have different sizes.
- The altitude is always perpendicular to the base or its extension.
- A median joins a vertex to the midpoint of the opposite side; it is not necessarily perpendicular to that side.

32. Quick Revision Table

Concept	Key Result
Triangle angle sum	180°
Exterior angle	Sum of two opposite interior angles

Equilateral triangle	Three equal sides; each angle 60°
Isosceles triangle	Two equal sides; opposite angles equal
Right triangle	One angle 90°
Pythagorean theorem	$H^2 = P^2 + B^2$
Triangle inequality	Sum of any two sides > third side
Centroid	Medians; ratio 2:1
Incentre	Angle bisectors
Circumcentre	Perpendicular bisectors
Orthocentre	Altitudes

33. Final Revision

Triangle angle sum = 180°

Exterior angle = Sum of opposite interior angles

Equilateral → 3 equal sides → 3 angles of 60°

Isosceles → 2 equal sides → 2 equal opposite angles

Right-angled → one angle of 90°

Pythagorean theorem → $H^2 = P^2 + B^2$

Congruence → Same shape and same size

Similarity → Same shape; corresponding sides proportional

The most effective way to solve triangle problems is to identify the type of triangle first, write the relevant property or theorem, and then substitute the given values.