



Chapter 5: Plane Geometry & Loci

Chapter 4 — Quadrilaterals | Detailed Notes & Solved Examples

1. What Is a Quadrilateral?

A quadrilateral is a closed plane figure made up of four line segments. It has four sides, four vertices and four interior angles.

$$\text{Sum of interior angles of a quadrilateral} = 360^\circ$$

The word quadrilateral simply means a figure with four sides. Different quadrilaterals are classified according to their sides, angles and diagonals.

2. Basic Parts of a Quadrilateral

The four line segments are its sides, the points where two sides meet are its vertices, and the line segments joining opposite vertices are called diagonals.

A quadrilateral has exactly two diagonals.

3. Main Types of Quadrilaterals

Quadrilateral	Main Property	Important Features
Parallelogram	Both pairs of opposite sides are parallel	Opposite sides and angles are equal; diagonals bisect each other
Rectangle	Four right angles	Opposite sides equal; diagonals equal and bisect each other
Square	Four equal sides and four right angles	All properties of a rectangle and rhombus
Rhombus	Four equal sides	Opposite angles equal; diagonals are perpendicular bisectors
Trapezium	One pair of opposite sides is parallel	The parallel sides are called bases
Kite	Two pairs of adjacent sides are equal	One pair of opposite angles is equal; diagonals have special properties

4. Angle Sum Property

The four interior angles of every quadrilateral add up to 360° .

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

One easy way to understand this is to draw a diagonal. The diagonal divides the quadrilateral into two triangles. Each triangle has an angle sum of 180° , so the total is 360° .

5. Diagonals of a Quadrilateral

A diagonal joins two opposite vertices of a quadrilateral. Every quadrilateral has two diagonals.

The behaviour of the diagonals depends on the type of quadrilateral.

6. Parallelogram

A parallelogram is a quadrilateral in which both pairs of opposite sides are parallel.

A parallelogram is one of the most important quadrilaterals because many other figures can be understood through its properties.

7. Properties of a Parallelogram

- Opposite sides are parallel.
- Opposite sides are equal.
- Opposite angles are equal.
- Adjacent angles are supplementary.
- The diagonals bisect each other.

Adjacent angles of a parallelogram add up to 180° .

8. Rectangle

A rectangle is a parallelogram with four right angles.

Each angle of a rectangle = 90°

Opposite sides are equal and parallel. The diagonals are equal and bisect each other.

9. Properties of a Rectangle

- Opposite sides are equal and parallel.
- All four angles are 90° .
- Diagonals are equal.
- Diagonals bisect each other.
- A rectangle is always a parallelogram.

10. Square

A square has four equal sides and four right angles. It combines the properties of a rectangle and a rhombus.

- All four sides are equal.
- All four angles are 90° .
- Opposite sides are parallel.
- Diagonals are equal.
- Diagonals bisect each other.
- Diagonals are perpendicular.
- Each diagonal bisects the opposite angles.

11. Rhombus

A rhombus is a parallelogram in which all four sides are equal.

- All four sides are equal.
- Opposite sides are parallel.
- Opposite angles are equal.



- Adjacent angles are supplementary.
- Diagonals bisect each other at right angles.
- Each diagonal bisects a pair of opposite angles.

12. Trapezium

A trapezium is a quadrilateral with one pair of opposite sides parallel. The parallel sides are called the bases.

The non-parallel sides are called the legs.

In an isosceles trapezium, the non-parallel sides are equal and the base angles are equal.

13. Kite

A kite is a quadrilateral with two pairs of adjacent equal sides.

A kite is different from a parallelogram because the equal sides are adjacent rather than opposite.

- Two pairs of adjacent sides are equal.
- One pair of opposite angles is equal.
- Its diagonals are perpendicular.
- One diagonal bisects the other.

14. Special Relationships

Every square is a rectangle and a rhombus. Every rectangle and every rhombus is a parallelogram.

Square $\longrightarrow$ Rectangle $\longrightarrow$ Parallelogram

Square $\longrightarrow$ Rhombus $\longrightarrow$ Parallelogram

The reverse statements are not generally true. For example, every rectangle is not a square because its four sides need not be equal.

15. Area of a Parallelogram

The area of a parallelogram is the product of its base and the corresponding perpendicular height.

$$\text{Area} = \text{Base} \times \text{Height}$$

The height is the perpendicular distance between the two parallel sides, not the slanting side.

16. Area of a Rectangle

$$\text{Area} = \text{Length} \times \text{Breadth}$$

$$\text{Perimeter} = 2(\text{Length} + \text{Breadth})$$

The diagonal of a rectangle can be found using Pythagoras:

$$\text{Diagonal}^2 = \text{Length}^2 + \text{Breadth}^2$$

17. Area of a Square

$$\text{Area} = \text{Side}^2$$



$$\text{Perimeter} = 4 \times \text{Side}$$

$$\text{Diagonal} = \text{Side} \times \sqrt{2}$$

18. Area of a Rhombus

The area of a rhombus can be found using its diagonals.

$$\text{Area} = 1/2 \times d_1 \times d_2$$

The two diagonals of a rhombus intersect at right angles and bisect each other.

19. Area of a Trapezium

If the parallel sides are a and b and the perpendicular distance between them is h :

$$\text{Area} = 1/2 \times (a + b) \times h$$

20. Area of a Kite

The area of a kite can also be calculated using its diagonals.

$$\text{Area} = 1/2 \times d_1 \times d_2$$

21. Perimeter

The perimeter of a quadrilateral is the total length of all four sides.

$$\text{Perimeter} = a + b + c + d$$

For special quadrilaterals, simpler formulas can be used.

22. Important Diagonal Properties

Figure	Equal Diagonals	Perpendicular Diagonals	Bisect Each Other
Parallelogram	Not always	Not always	Yes
Rectangle	Yes	Not generally	Yes
Square	Yes	Yes	Yes
Rhombus	Not generally	Yes	Yes
Kite	Not generally	Yes	One diagonal bisects the other

23. Detailed Solved Examples

Example 1

Three angles of a quadrilateral are 80° , 95° and 105° . Find the fourth angle.

The sum of the angles of a quadrilateral is 360° .

$$80^\circ + 95^\circ + 105^\circ + x = 360^\circ.$$

$$280^\circ + x = 360^\circ.$$

$$\text{Therefore } x = 80^\circ.$$

Answer: 80°



Example 2

The angles of a quadrilateral are in the ratio 2:3:4:6. Find all four angles.

Let the angles be $2x^\circ$, $3x^\circ$, $4x^\circ$ and $6x^\circ$.

Their sum is 360° .

$$2x + 3x + 4x + 6x = 360^\circ.$$

$$15x = 360^\circ.$$

$$x = 24^\circ.$$

The angles are 48° , 72° , 96° and 144° .

Answer: 48° , 72° , 96° and 144°

Example 3

One angle of a parallelogram is 65° . Find all its angles.

Opposite angles of a parallelogram are equal.

Adjacent angles are supplementary.

Therefore the opposite angle is 65° and each adjacent angle is $180^\circ - 65^\circ = 115^\circ$.

Answer: 65° , 115° , 65° , 115°

Example 4

Adjacent angles of a parallelogram are $(3x + 10)^\circ$ and $(2x + 20)^\circ$. Find x .

Adjacent angles of a parallelogram are supplementary.

$$(3x + 10) + (2x + 20) = 180.$$

$$5x + 30 = 180.$$

$$5x = 150.$$

$$\text{Therefore } x = 30.$$

Answer: $x = 30$

Example 5

The base of a parallelogram is 15 cm and its perpendicular height is 8 cm. Find its area.

$$\text{Area} = \text{Base} \times \text{Height}.$$

$$\text{Area} = 15 \times 8.$$

Therefore the area is 120 cm^2 .

Answer: 120 cm^2

Example 6

A rectangle is 12 cm long and 5 cm wide. Find its area and perimeter.

$$\text{Area} = \text{Length} \times \text{Breadth} = 12 \times 5 = 60 \text{ cm}^2.$$

$$\text{Perimeter} = 2(\text{Length} + \text{Breadth}) = 2(12 + 5) = 34 \text{ cm}.$$



Answer: Area = 60 cm²; Perimeter = 34 cm

Example 7

A rectangle has length 9 cm and breadth 12 cm. Find its diagonal.

Use Pythagoras: $d^2 = l^2 + b^2$.

$$d^2 = 9^2 + 12^2 = 81 + 144 = 225.$$

Therefore $d = 15$ cm.

Answer: 15 cm

Example 8

The side of a square is 8 cm. Find its area, perimeter and diagonal.

$$\text{Area} = 8^2 = 64 \text{ cm}^2.$$

$$\text{Perimeter} = 4 \times 8 = 32 \text{ cm}.$$

$$\text{Diagonal} = 8\sqrt{2} \text{ cm}.$$

Answer: Area = 64 cm²; Perimeter = 32 cm; Diagonal = $8\sqrt{2}$ cm

Example 9

The diagonals of a rhombus are 16 cm and 12 cm. Find its area.

$$\text{Area} = \frac{1}{2} \times d_1 \times d_2.$$

$$\text{Area} = \frac{1}{2} \times 16 \times 12.$$

Therefore the area is 96 cm².

Answer: 96 cm²

Example 10

A rhombus has diagonals 10 cm and 24 cm. Find its area.

$$\text{Area} = \frac{1}{2} \times 10 \times 24.$$

$$\text{Area} = 120 \text{ cm}^2.$$

Answer: 120 cm²

Example 11

The parallel sides of a trapezium are 12 cm and 18 cm, and its height is 7 cm. Find the area.

$$\text{Area} = \frac{1}{2} \times (a + b) \times h.$$

$$\text{Area} = \frac{1}{2} \times (12 + 18) \times 7.$$

$$= \frac{1}{2} \times 30 \times 7 = 105 \text{ cm}^2.$$

Answer: 105 cm²

Example 12

The diagonals of a kite are 14 cm and 10 cm. Find its area.

$$\text{Area} = \frac{1}{2} \times d_1 \times d_2.$$

$$\text{Area} = \frac{1}{2} \times 14 \times 10.$$

Therefore the area is 70 cm².

Answer: 70 cm²

Example 13

A square has an area of 144 cm². Find its side and perimeter.

$$\text{Area} = \text{side}^2.$$

$$\text{side}^2 = 144, \text{ so side} = 12 \text{ cm.}$$

$$\text{Perimeter} = 4 \times 12 = 48 \text{ cm.}$$

Answer: Side = 12 cm; Perimeter = 48 cm

Example 14

A rectangle has area 96 cm² and length 12 cm. Find its breadth.

$$\text{Area} = \text{Length} \times \text{Breadth.}$$

$$96 = 12 \times \text{breadth.}$$

$$\text{Breadth} = 8 \text{ cm.}$$

Answer: 8 cm

Example 15

A parallelogram has area 180 cm² and base 15 cm. Find its perpendicular height.

$$\text{Area} = \text{Base} \times \text{Height.}$$

$$180 = 15 \times h.$$

$$h = 12 \text{ cm.}$$

Answer: 12 cm

Example 16

A trapezium has area 120 cm², parallel sides 10 cm and 14 cm. Find its height.

$$120 = \frac{1}{2} \times (10 + 14) \times h.$$

$$120 = 12h.$$

$$\text{Therefore } h = 10 \text{ cm.}$$

Answer: 10 cm

Example 17

The diagonals of a square are each 10 cm. Find its area.

$$\text{For a square, Area} = \frac{1}{2} \times d^2.$$



$$\text{Area} = \frac{1}{2} \times 10^2.$$

$$\text{Area} = 50 \text{ cm}^2.$$

Answer: 50 cm²

Example 18

A quadrilateral has three angles 70°, 80° and 110°. Find the fourth angle.

$$\text{Angle sum} = 360^\circ.$$

$$70 + 80 + 110 + x = 360.$$

$$260 + x = 360.$$

$$\text{Therefore } x = 100^\circ.$$

Answer: 100°

24. How to Identify a Quadrilateral

When you see a quadrilateral in a question, first look for parallel sides, equal sides, right angles and diagonal properties.

- If both pairs of opposite sides are parallel → parallelogram.
- If all four angles are 90° → rectangle; if the four sides are also equal → square.
- If all four sides are equal → rhombus; if its angles are also 90° → square.
- If only one pair of opposite sides is parallel → trapezium.
- If two pairs of adjacent sides are equal → kite.

25. Common Mistakes

- Do not use the slanting side of a parallelogram as its height. Height means perpendicular distance between the parallel sides.
- Do not assume every rectangle is a square. A rectangle needs four right angles, while a square also requires all four sides to be equal.
- Do not assume the diagonals of every quadrilateral are equal or perpendicular.
- Remember that the diagonals of a parallelogram bisect each other, but they are not necessarily equal.
- For a rhombus, the diagonals are perpendicular, but they are not necessarily equal.
- Use the correct pair of parallel sides when applying the trapezium area formula.

26. Quick Revision Table

Figure	Main Properties	Area
Parallelogram	Opposite sides equal and parallel; diagonals bisect each other	$b \times h$
Rectangle	Four right angles; diagonals equal	$l \times b$
Square	Four equal sides; four right angles	a^2
Rhombus	Four equal sides; perpendicular diagonals	$\frac{1}{2} d_1 d_2$
Trapezium	One pair of opposite sides parallel	$\frac{1}{2}(a+b)h$
Kite	Two pairs of adjacent equal sides; perpendicular diagonals	$\frac{1}{2} d_1 d_2$



27. Final Formula Sheet

$$\text{Quadrilateral angle sum} = 360^\circ$$

$$\text{Parallelogram area} = \text{Base} \times \text{Height}$$

$$\text{Rectangle area} = \text{Length} \times \text{Breadth}$$

$$\text{Rectangle perimeter} = 2(\text{Length} + \text{Breadth})$$

$$\text{Rectangle diagonal}^2 = \text{Length}^2 + \text{Breadth}^2$$

$$\text{Square area} = \text{Side}^2$$

$$\text{Square perimeter} = 4 \times \text{Side}$$

$$\text{Square diagonal} = \text{Side} \times \sqrt{2}$$

$$\text{Rhombus area} = 1/2 \times d_1 \times d_2$$

$$\text{Trapezium area} = 1/2 \times (a + b) \times h$$

$$\text{Kite area} = 1/2 \times d_1 \times d_2$$

The key to quadrilateral problems is to identify the figure first. Once its defining properties are known, choose the appropriate angle, side, diagonal, perimeter or area formula.