Chapter 5: Plane Geometry & Loci

Chapter 5 — Circles | Detailed Notes & Solved Examples

1. What Is a Circle?

A circle is a closed plane curve in which every point is at the same distance from a fixed point. The fixed point is called the centre.

Distance from centre to every point on the circle = Radius

A circle has no sides or vertices. It is completely described by its centre and radius.

2. Basic Terms of a Circle

Term	Meaning	Important Point
Centre	Fixed point inside the circle	Usually denoted by O
Radius	Line segment from centre to any point on circle	All radii are equal
Diameter	Chord passing through the centre	Diameter = 2 × radius
Chord	Line segment joining two points on circle	Longest chord is diameter
Arc	Part of the circumference	Can be minor or major
Circumference	Complete boundary of circle	$2\pi r$
Sector	Region bounded by two radii and an arc	Like a slice of a circle
Segment	Region bounded by a chord and an arc	Minor or major segment
Tangent	Line touching circle at exactly one point	Perpendicular to radius at contact point
Secant	Line cutting circle at two points	Passes through the circle

3. Radius

A radius is a line segment joining the centre of the circle to any point on its circumference.

All radii of the same circle are equal.

If the radius is r, then the diameter is twice the radius.

4. Diameter

A diameter is a chord that passes through the centre of the circle.

Diameter = 2r

Radius = Diameter/2

The diameter is the longest possible chord of a circle.

5. Chord

A chord is a line segment whose endpoints lie on the circle. Every diameter is a chord, but every chord is not a diameter.

A chord passing through the centre is always the diameter.

6. Arc

An arc is a part of the circumference of a circle between two points.

A minor arc is smaller than a semicircle, while a major arc is larger than a semicircle. A semicircle is an arc measuring 180°.

7. Circumference

The circumference is the total distance around a circle.

Circumference = $2\pi r$

Circumference = πd

For most school calculations, π may be taken as 22/7 or 3.14, depending on the question.

8. Area of a Circle

The area of a circle is the region enclosed by its circumference.

Area = πr^2

The area depends on the square of the radius. Therefore, doubling the radius makes the area four times as large.

9. Semicircle

A semicircle is half of a circle.

Area of semicircle = $1/2\ \pi r^2$

Length of curved part = πr

If the perimeter of a semicircle is required, remember to include the diameter as well:

Perimeter of semicircle = $\pi r + 2r$

10. Sector

A sector is the region enclosed by two radii and the arc between them. A sector is often described by its central angle.

Area of sector = $\theta/360° \times \pi r^2$

Arc length = $\theta/360° \times 2\pi r$

For a semicircle, $\theta = 180°$. For a quadrant, $\theta = 90°$.

11. Quadrant

A quadrant is one-fourth of a circle. Its central angle is 90°.

Area of quadrant = $1/4\ \pi r^2$

Arc length of quadrant = $1/4 \times 2\pi r = \pi r/2$

12. Tangent to a Circle

A tangent is a straight line that touches a circle at exactly one point. That point is called the point of contact.

A tangent does not pass through the circle. It only touches the circle at the point of contact.

13. Radius and Tangent

The radius drawn from the centre to the point of contact is perpendicular to the tangent.

Radius ⊥ Tangent

Angle between radius and tangent = 90°

This property is extremely useful in geometry problems involving tangents.

14. Tangents From an External Point

If two tangents are drawn from the same external point to a circle, the lengths of the two tangent segments are equal.

PA = PB

Here P is the external point and A and B are the points of contact.

15. Angle at the Centre and Angle at the Circumference

An angle subtended by an arc at the centre of a circle is twice the angle subtended by the same arc at any point on the remaining circumference.

Angle at centre = 2 × Angle at circumference

16. Angle in a Semicircle

An angle subtended by a diameter at any point on the circle is a right angle.

Angle in a semicircle = 90°

This is a very useful theorem. Whenever a triangle is formed using a diameter as one side and a point on the circle as the third vertex, the angle at that third vertex is 90°.

17. Equal Chords

Equal chords of the same circle are equidistant from the centre.

Conversely, chords that are at the same distance from the centre are equal.

18. Perpendicular From Centre to a Chord

The perpendicular drawn from the centre of a circle to a chord bisects the chord.

Perpendicular from centre to chord → Bisects chord

This means that if OM is perpendicular to chord AB, then AM = MB.

19. Equal Chords and Equal Angles

Equal chords of the same circle subtend equal angles at the centre.

Equal chords → Equal central angles

Equal chords also subtend equal angles at points on the remaining part of the circle.

20. Cyclic Quadrilateral

A cyclic quadrilateral is a quadrilateral whose four vertices lie on the same circle.

The opposite angles of a cyclic quadrilateral are supplementary.

Opposite angles of a cyclic quadrilateral add up to 180°

21. Important Circle Theorems

• The diameter is the longest chord of a circle.

• The radius is perpendicular to the tangent at the point of contact.

• Tangents drawn from the same external point are equal in length.

• The angle in a semicircle is 90°.

• The angle at the centre is twice the angle at the circumference standing on the same arc.

• A perpendicular from the centre to a chord bisects the chord.

• Equal chords are equidistant from the centre.

• Equal chords subtend equal angles at the centre.

• Opposite angles of a cyclic quadrilateral are supplementary.

22. Detailed Solved Examples

Example 1

A circle has radius 7 cm. Find its diameter.

Diameter = 2 × radius.

Diameter = 2 × 7 = 14 cm.

Answer: 14 cm

Example 2

A circle has diameter 20 cm. Find its radius.

Radius = Diameter/2.

Radius = 20/2 = 10 cm.

Answer: 10 cm

Example 3

Find the circumference of a circle with radius 7 cm. Take π = 22/7.

Circumference = 2πr.

= 2 × 22/7 × 7.

= 44 cm.

Answer: 44 cm

Example 4

Find the area of a circle with radius 7 cm. Take $\pi = 22/7$.

Area = πr^2.

= $22/7 \times 7^2$.

= $22 \times 7 = 154$ cm².

Answer: 154 cm²

Example 5

The circumference of a circle is 44 cm. Find its radius. Take $\pi = 22/7$.

$2\pi r = 44$.

$2 \times 22/7 \times r = 44$.

$r = 7$ cm.

Answer: 7 cm

Example 6

The area of a circle is 616 cm². Find its radius. Take $\pi = 22/7$.

$\pi r^2 = 616$.

$22/7 \times r^2 = 616$.

$r^2 = 616 \times 7/22 = 196$.

Therefore $r = 14$ cm.

Answer: 14 cm

Example 7

Find the area of a semicircle of radius 14 cm. Take $\pi = 22/7$.

Area of semicircle = $1/2\ \pi r^2$.

= $1/2 \times 22/7 \times 14^2$.

= 308 cm².

Answer: 308 cm²

Example 8

Find the perimeter of a semicircle of radius 7 cm. Take $\pi = 22/7$.

Perimeter = curved length + diameter.

Curved length = πr = 22 cm.

Diameter = 14 cm.

Total perimeter = 22 + 14 = 36 cm.

Answer: 36 cm

Example 9

Find the area of a quadrant of radius 14 cm. Take $\pi = 22/7$.

Area of quadrant = $1/4\ \pi r^2$.

= $1/4 \times 22/7 \times 196$.

= $154\ cm^2$.

Answer: $154\ cm^2$

Example 10

Find the area of a sector with radius 14 cm and central angle 90°. Take $\pi = 22/7$.

Area = $\theta/360° \times \pi r^2$.

= $90/360 \times 22/7 \times 196$.

= $1/4 \times 616 = 154\ cm^2$.

Answer: $154\ cm^2$

Example 11

Find the length of an arc of a circle with radius 21 cm and central angle 60°. Take $\pi = 22/7$.

Arc length = $\theta/360° \times 2\pi r$.

= $60/360 \times 2 \times 22/7 \times 21$.

= 22 cm.

Answer: 22 cm

Example 12

A tangent touches a circle at P. If O is the centre, what is the angle between OP and the tangent?

The radius drawn to the point of contact is perpendicular to the tangent.

Therefore the angle is 90°.

Answer: 90°

Example 13

Two tangents PA and PB are drawn from an external point P to a circle. If PA = 8 cm, find PB.

Tangents from the same external point are equal.

Therefore PB = PA = 8 cm.

Answer: 8 cm

Example 14

A chord AB of a circle is perpendicular to the radius OM at M. If AB = 18 cm, find AM and MB.

A perpendicular from the centre to a chord bisects the chord.

Therefore AM = MB.

Each half = 18/2 = 9 cm.

Answer: AM = 9 cm and MB = 9 cm

Example 15

An angle at the circumference standing on an arc is 35°. Find the angle at the centre standing on the same arc.

Angle at centre = 2 × angle at circumference.

= 2 × 35° = 70°.

Answer: 70°

Example 16

The angle at the centre standing on an arc is 120°. Find the angle at the circumference standing on the same arc.

Angle at circumference = half the angle at centre.

= 120°/2 = 60°.

Answer: 60°

Example 17

AB is a diameter of a circle and C is a point on the circle. Find ∠ACB.

An angle subtended by a diameter at the circumference is a right angle.

Therefore ∠ACB = 90°.

Answer: 90°

Example 18

A cyclic quadrilateral has one angle of 75°. Find the opposite angle.

Opposite angles of a cyclic quadrilateral are supplementary.

Opposite angle = 180° − 75° = 105°.

Answer: 105°

Example 19

A cyclic quadrilateral has opposite angles (2x + 10)° and (3x − 20)°. Find x.

Opposite angles are supplementary.

(2x + 10) + (3x − 20) = 180.

5x − 10 = 180.

5x = 190.

x = 38.

Answer: x = 38

Example 20

A circle has radius 10 cm. Find its circumference and area in terms of π.

Circumference = $2\pi r = 2\pi \times 10 = 20\pi$ cm.

Area = $\pi r^2 = \pi \times 10^2 = 100\pi$ cm^2.

Answer: Circumference = 20π cm; Area = 100π cm^2

23. How to Solve Circle Problems

• Identify what is given: radius, diameter, chord, arc, sector, tangent or angle.

• If diameter is given, first convert it into radius using $r = d/2$ when needed.

• Choose the correct formula for circumference, area, arc length or sector area.

• For tangent questions, remember that the radius to the point of contact makes 90° with the tangent.

• For angle questions, check whether the problem involves a diameter, the same arc, equal chords or a cyclic quadrilateral.

• Keep units consistent and write square units for area.

24. Common Mistakes

• Do not confuse radius with diameter. Diameter is twice the radius.

• Do not use $2\pi r$ for area. $2\pi r$ gives circumference, while πr^2 gives area.

• Remember that the perimeter of a semicircle includes the diameter if the complete boundary is required.

• A tangent touches a circle at one point; a secant cuts it at two points.

• Do not forget the factor θ/360° in sector area and arc-length formulas.

• The angle in a semicircle is 90°, not 180°.

• For a cyclic quadrilateral, opposite angles—not adjacent angles—are supplementary.

25. Quick Revision Table

Concept	Key Result
Diameter	$2r$
Circumference	$2\pi r$
Area of circle	πr^2
Area of semicircle	$1/2\ \pi r^2$
Area of sector	$\theta/360° \times \pi r^2$
Arc length	$\theta/360° \times 2\pi r$
Radius and tangent	Perpendicular at point of contact
Tangents from same external point	Equal
Angle in semicircle	90°
Centre angle and circumference angle	Centre angle = 2 × circumference angle
Cyclic quadrilateral	Opposite angles sum to 180°

26. Final Formula Sheet

Diameter = $2r$

Circumference = $2\pi r = \pi d$

Area = πr^2

Semicircle area = $1/2\ \pi r^2$

Semicircle perimeter = $\pi r + 2r$

Sector area = $\theta/360° \times \pi r^2$

Arc length = $\theta/360° \times 2\pi r$

Quadrant area = $1/4\ \pi r^2$

Radius $\perp$ Tangent

Angle at centre = 2 × angle at circumference

Angle in a semicircle = 90°

Opposite angles of cyclic quadrilateral = 180°

The easiest way to handle circle questions is to first identify the part of the circle involved—radius, diameter, chord, tangent, sector or arc. Then apply the corresponding property or formula.