Chapter 6: Mensuration (2D & 3D)

Chapter 1 — 2D Areas | Detailed Notes & Solved Examples

1. Introduction to Mensuration

Mensuration is the branch of mathematics that deals with the measurement of geometrical figures.

In two-dimensional geometry, we mainly calculate perimeter and area. Perimeter is the total length of the boundary, while area is the region covered by a closed figure.

Common two-dimensional figures include square, rectangle, triangle, parallelogram, rhombus, trapezium, circle, sector and quadrant.

2. Basic Units of Measurement

Length is measured in units such as metre, centimetre and kilometre. Area is measured in square units.

$$1 \text{ m} = 100 \text{ cm}$$

$$1 \text{ km} = 1000 \text{ m}$$

$$1 \text{ m}^2 = 10{,}000 \text{ cm}^2$$

$$1 \text{ km}^2 = 10{,}00{,}000 \text{ m}^2$$

$$1 \text{ hectare} = 10{,}000 \text{ m}^2$$

Always convert all measurements into the same unit before calculating an area.

3. Rectangle

A rectangle is a quadrilateral in which all four angles are right angles and opposite sides are equal.

$$\text{Area} = \text{Length} \times \text{Breadth}$$

$$\text{Perimeter} = 2(\text{Length} + \text{Breadth})$$

$$\text{Diagonal} = \sqrt{(\text{Length}^2 + \text{Breadth}^2)}$$

Example 1

A rectangular field is 50 m long and 30 m wide. Find its area.

Area = Length × Breadth.

Area = 50 × 30 = 1500 m^2.

Answer: 1500 m^2

Example 2

The length of a rectangle is 20 cm and its breadth is 12 cm. Find its perimeter.

Perimeter = 2(Length + Breadth).

P = 2(20 + 12) = 64 cm.

Answer: 64 cm

4. Square

A square has four equal sides and four right angles. If its side is a:

$$\text{Area} = a^2$$

$$\text{Perimeter} = 4a$$

$$\text{Diagonal} = a\sqrt{2}$$

$$\text{Area in terms of diagonal} = d^2/2$$

Example 3

The side of a square is 14 cm. Find its area and perimeter.

Area = $14^2 = 196$ cm^2.

Perimeter = $4 \times 14 = 56$ cm.

Answer: Area = 196 cm^2; Perimeter = 56 cm

5. Triangle

A triangle has three sides and three angles. Its area is half the product of its base and perpendicular height.

$$\text{Area} = 1/2 \times \text{Base} \times \text{Height}$$

The height used in this formula must be perpendicular to the chosen base.

Example 4

The base of a triangle is 16 cm and its perpendicular height is 9 cm. Find its area.

Area = $1/2 \times 16 \times 9$.

Area = 72 cm^2.

Answer: 72 cm^2

6. Equilateral Triangle

In an equilateral triangle, all three sides are equal.

$$\text{Area} = (\sqrt{3}/4)a^2$$

$$\text{Height} = (\sqrt{3}/2)a$$

$$\text{Perimeter} = 3a$$

Example 5

Find the area of an equilateral triangle whose side is 8 cm.

Area = $(\sqrt{3}/4) \times 8^2$.

Area = $16\sqrt{3}$ cm^2.

Answer: $16\sqrt{3}$ cm^2

7. Heron's Formula

When all three sides of a triangle are known, but the height is not given, Heron's formula can be used.

$$s = (a + b + c)/2$$

$$\text{Area} = \sqrt{s(s - a)(s - b)(s - c)}$$

Example 6

Find the area of a triangle whose sides are 5 cm, 6 cm and 7 cm.

$s = (5 + 6 + 7)/2 = 9$.

Area = $\sqrt{9(9-5)(9-6)(9-7)}$.

Area = $\sqrt{216} = 6\sqrt{6}$ cm^2.

Answer: $6\sqrt{6}$ cm^2

8. Parallelogram

A parallelogram has two pairs of opposite parallel sides.

$$\text{Area} = \text{Base} \times \text{Perpendicular Height}$$

$$\text{Perimeter} = 2(a + b)$$

The slanting side is not necessarily the height. Height means the perpendicular distance between the parallel sides.

Example 7

A parallelogram has a base of 18 cm and height of 7 cm. Find its area.

Area = 18×7.

Area = 126 cm^2.

Answer: 126 cm^2

9. Rhombus

A rhombus has four equal sides. Its diagonals bisect each other at right angles.

$$\text{Area} = 1/2 \times d_1 \times d_2$$

$$\text{Perimeter} = 4a$$

Example 8

The diagonals of a rhombus are 16 cm and 12 cm. Find its area.

Area = $1/2 \times 16 \times 12$.

Area = 96 cm^2.

Answer: 96 cm^2

10. Trapezium

A trapezium has one pair of opposite sides parallel. If the parallel sides are a and b and the height is h:

$$\text{Area} = 1/2 \times (a + b) \times h$$

Example 9

The parallel sides of a trapezium are 12 cm and 18 cm. Its height is 8 cm. Find the area.

Area = $1/2 \times (12 + 18) \times 8$.

Area = 120 cm^2.

Answer: 120 cm^2

11. Kite

A kite has two pairs of adjacent equal sides.

$$\text{Area} = 1/2 \times d_1 \times d_2$$

Example 10

The diagonals of a kite are 20 cm and 12 cm. Find its area.

Area = $1/2 \times 20 \times 12$.

Area = 120 cm^2.

Answer: 120 cm^2

12. Circle

A circle is the set of all points that are at the same distance from a fixed point called the centre.

$$\text{Circumference} = 2\pi r = \pi d$$

$$\text{Area} = \pi r^2$$

Unless another value is specified, use $\pi = 22/7$ or $\pi = 3.14$ as appropriate.

Example 11

Find the area of a circle of radius 7 cm. Take $\pi = 22/7$.

Area = πr^2.

Area = $22/7 \times 7^2 = 154$ cm^2.

Answer: 154 cm^2

Example 12

A circular park has a diameter of 28 m. Find its area.

Radius = $28/2 = 14$ m.

Area = $22/7 \times 14^2 = 616$ m^2.

Answer: 616 m^2

13. Semicircle

A semicircle is half of a circle.

$$\text{Area} = 1/2\ \pi r^2$$

$$\text{Curved length} = \pi r$$

$$\text{Perimeter} = \pi r + 2r$$

Example 13

Find the area of a semicircle of radius 14 cm. Take $\pi = 22/7$.

Area = $1/2 \times 22/7 \times 14^2$.

Area = $308\ cm^2$.

Answer: $308\ cm^2$

14. Quadrant

A quadrant is one-fourth of a circle and has a central angle of 90°.

$$\text{Area} = 1/4\ \pi r^2$$

$$\text{Arc length} = \pi r/2$$

15. Sector

A sector is the region bounded by two radii and the arc between them.

$$\text{Area of sector} = \theta/360° \times \pi r^2$$

$$\text{Arc length} = \theta/360° \times 2\pi r$$

Example 14

Find the area of a sector with radius 14 cm and central angle 90°. Take $\pi = 22/7$.

Area = $90/360 \times 22/7 \times 14^2$.

Area = $154\ cm^2$.

Answer: $154\ cm^2$

16. Segment of a Circle

A segment is the region bounded by a chord and its corresponding arc.

For a minor segment:

$$\text{Area of segment} = \text{Area of sector} - \text{Area of triangle}$$

Do not confuse a sector with a segment. A sector has two radii, whereas a segment has a chord.

17. Composite Figures

A composite figure is made by combining two or more simple shapes. Break the figure into familiar shapes and calculate each area separately.

If a part is removed, subtract its area. If a part is added, add its area.

Example 15

A rectangle is 20 cm long and 10 cm wide. A semicircle of diameter 10 cm is attached to one side. Find the total area.

Rectangle area = $20 \times 10 = 200\ cm^2$.

Semicircle radius = 5 cm.

Semicircle area = $1/2 \times \pi \times 5^2 = 25\pi/2\ cm^2$.

Total area = $200 + 25\pi/2$ cm².

Answer: $200 + 25\pi/2$ cm²

18. Area of a Path Around a Figure

For a path surrounding a figure, calculate the outer area and subtract the inner area.

Area of path = Outer area − Inner area

Example 16

A rectangular garden is 20 m × 15 m. A path 2 m wide surrounds it externally. Find the area of the path.

Outer dimensions = (20 + 4) m × (15 + 4) m = 24 m × 19 m.

Outer area = 456 m².

Garden area = 300 m².

Area of path = 456 − 300 = 156 m².

Answer: 156 m²

19. Important Area Relationships

Rectangle → lb

Square → a^2

Triangle → $1/2\ bh$

Equilateral triangle → $\sqrt{3}/4\ a^2$

Parallelogram → bh

Rhombus → $1/2\ d_1d_2$

Trapezium → $1/2(a+b)h$

Circle → πr^2

Semicircle → $1/2\pi r^2$

Sector → $\theta/360° \times \pi r^2$

20. More Solved Examples

Example 17

The area of a rectangle is 480 cm² and its length is 24 cm. Find its breadth.

Area = Length × Breadth.

480 = 24 × breadth.

Breadth = 20 cm.

Answer: 20 cm

Example 18

The area of a square is 625 cm². Find its side.

$a^2 = 625$.

$a = 25$ cm.

Answer: 25 cm

Example 19

The base of a triangle is 24 cm and its area is 180 cm². Find its height.

$180 = 1/2 \times 24 \times h$.

$180 = 12h$.

$h = 15$ cm.

Answer: 15 cm

Example 20

The diagonals of a rhombus are 18 cm and 24 cm. Find its area.

Area = $1/2 \times 18 \times 24$.

Area = 216 cm^2.

Answer: 216 cm^2

Example 21

A trapezium has parallel sides 15 cm and 25 cm and height 10 cm. Find its area.

Area = $1/2 \times (15 + 25) \times 10$.

Area = 200 cm^2.

Answer: 200 cm^2

Example 22

Find the circumference and area of a circle of radius 14 cm. Take $\pi = 22/7$.

Circumference = $2 \times 22/7 \times 14 = 88$ cm.

Area = $22/7 \times 14^2 = 616$ cm^2.

Answer: Circumference = 88 cm; Area = 616 cm^2

21. Effect of Changing Dimensions

Understanding how area changes when dimensions change can save time in calculation-based questions.

If the radius of a circle doubles → area becomes 4 times.

If the radius becomes 3 times → area becomes 9 times.

If the side of a square doubles → area becomes 4 times.

For a rectangle, if only the length doubles while breadth remains unchanged, the area also doubles.

22. Important Exam Points

• Perimeter measures the boundary; area measures the enclosed region.

- Area is always expressed in square units.
- Use the perpendicular height in triangle and parallelogram questions.
- Convert diameter into radius before using πr^2.
- For a semicircle's complete perimeter, include the diameter.
- For a sector, use $\theta/360°$.
- In a composite figure, split the figure into simple shapes.
- For a path, subtract inner area from outer area when the path lies around the figure.

23. Common Mistakes

- Do not confuse area with perimeter.
- Do not mix metres and centimetres in the same calculation.
- Do not use diameter as radius.
- Do not use a slanting side as the height of a parallelogram.
- Do not forget square units for area.
- Do not forget to include the diameter when finding the complete perimeter of a semicircle.
- Do not add an area that has been removed from a composite figure.

24. Quick Revision Table

Figure	Area	Perimeter
Square	a^2	4a
Rectangle	lb	2(l+b)
Triangle	1/2 bh	Sum of three sides
Equilateral Triangle	$\sqrt{3}/4\ a^2$	3a
Parallelogram	bh	2(a+b)
Rhombus	$1/2\ d_1d_2$	4a
Trapezium	1/2(a+b)h	Sum of four sides
Circle	πr^2	$2\pi r$
Semicircle	$1/2\pi r^2$	$\pi r + 2r$
Sector	$\theta/360 \times \pi r^2$	Arc + two radii

25. Final Formula Sheet

Rectangle: Area = lb

Square: Area = a^2

Triangle: Area = 1/2 bh

Equilateral triangle: Area = $\sqrt{3}/4\ a^2$

Parallelogram: Area = bh

Rhombus: Area = $1/2\ d_1d_2$

Trapezium: Area = 1/2(a+b)h

Circle: Area = πr^2

Circle: Circumference = $2\pi r$

Semicircle: Area = $1/2\pi r^2$

Sector: Area = $\theta/360° \times \pi r^2$

Arc length = $\theta/360° \times 2\pi r$

26. Final Revision

The key to mensuration is to identify the figure first and then select the correct formula. Keep all dimensions in the same unit, use perpendicular height wherever required, and check whether the question asks for area, perimeter or a combination of both.

Remember: perimeter is measured in units such as cm or m, while area is measured in square units such as cm^2 or m^2.