Chapter 6: Mensuration (2D & 3D)

Chapter 3 — 3D Surface Areas & Volumes | Detailed Notes & Solved Examples

1. Introduction to 3D Mensuration

Three-dimensional mensuration deals with solid objects that have length, breadth and height. Unlike a two-dimensional figure, a solid occupies space.

The two most important measurements are surface area and volume.

Surface area tells us how much outer surface a solid has. Volume tells us how much space the solid occupies.

2. Surface Area and Volume

Surface area → measured in square units

Volume → measured in cubic units

For example, surface area may be expressed in cm^2 or m^2, while volume may be expressed in cm^3 or m^3.

3. Important 3D Solids

The main solids used in mensuration are:

- Cuboid
- Cube
- Cylinder
- Cone
- Sphere
- Hemisphere
- Prism

4. Cuboid

A cuboid is a solid with six rectangular faces. A box, brick and rectangular room are common examples.

Let length = l, breadth = b and height = h.

$$\text{Volume} = lbh$$

$$\text{Total Surface Area} = 2(lb + bh + hl)$$

$$\text{Lateral Surface Area} = 2h(l + b)$$

$$\text{Diagonal} = \sqrt{l^2 + b^2 + h^2}$$

Example 1

A cuboid is 12 cm long, 8 cm broad and 5 cm high. Find its volume.

Volume = lbh.

$= 12 \times 8 \times 5$.

$= 480$ cm^3.

Answer: 480 cm^3

Example 2

Find the total surface area of a cuboid measuring 10 cm × 6 cm × 4 cm.

TSA $= 2(lb + bh + hl)$.

$= 2(10\times6 + 6\times4 + 4\times10)$.

$= 2(60 + 24 + 40) = 248$ cm^2.

Answer: 248 cm^2

5. Cube

A cube is a cuboid in which all edges are equal. Let the edge be a.

Volume = a^3

Total Surface Area = $6a^2$

Lateral Surface Area = $4a^2$

Diagonal = $a\sqrt{3}$

Example 3

Find the volume and total surface area of a cube of side 6 cm.

Volume $= 6^3 = 216$ cm^3.

TSA $= 6 \times 6^2 = 216$ cm^2.

Answer: Volume = 216 cm^3; TSA = 216 cm^2

6. Cylinder

A cylinder has two equal circular bases and a curved surface. Let radius = r and height = h.

Volume = $\pi r^2 h$

Curved Surface Area = $2\pi rh$

Total Surface Area = $2\pi r(h + r)$

The total surface area includes the two circular bases as well as the curved surface.

Example 4

A cylinder has radius 7 cm and height 10 cm. Find its volume. Take $\pi = 22/7$.

Volume $= \pi r^2 h$.

$= 22/7 \times 7^2 \times 10$.

$= 1540\ cm^3$.

Answer: $1540\ cm^3$

Example 5

Find the curved surface area of a cylinder with radius 7 cm and height 8 cm. Take $\pi = 22/7$.

CSA = $2\pi rh$.

$= 2 \times 22/7 \times 7 \times 8$.

$= 352\ cm^2$.

Answer: $352\ cm^2$

7. Cone

A cone has one circular base and a curved surface that narrows to a single point called the vertex.

Let radius = r, height = h and slant height = l.

$$\text{Slant height } l = \sqrt{(r^2 + h^2)}$$

$$\text{Volume} = 1/3\ \pi r^2 h$$

$$\text{Curved Surface Area} = \pi rl$$

$$\text{Total Surface Area} = \pi r(l + r)$$

Example 6

A cone has radius 6 cm and height 8 cm. Find its slant height.

$l = \sqrt{(r^2 + h^2)}$.

$= \sqrt{(6^2 + 8^2)}$.

$= \sqrt{100} = 10$ cm.

Answer: 10 cm

Example 7

Find the volume of a cone with radius 6 cm and height 8 cm. Take $\pi = 22/7$.

Volume = $1/3\ \pi r^2 h$.

$= 1/3 \times 22/7 \times 36 \times 8$.

$= 2112/7\ cm^3$.

Approximately $301.71\ cm^3$.

Answer: $2112/7\ cm^3$ (approximately $301.71\ cm^3$)

8. Sphere

A sphere is a perfectly round solid in which every point on its surface is at the same distance from the centre.

$$\text{Surface Area} = 4\pi r^2$$

$$\text{Volume} = 4/3\ \pi r^3$$

Example 8

Find the surface area of a sphere of radius 7 cm. Take π = 22/7.

Surface area = $4\pi r^2$.

= 4 × 22/7 × 49.

= 616 cm^2.

Answer: 616 cm^2

Example 9

Find the volume of a sphere of radius 3 cm in terms of π.

Volume = $4/3\ \pi r^3$.

= 4/3 × π × 27.

= 36π cm^3.

Answer: 36π cm^3

9. Hemisphere

A hemisphere is exactly half of a sphere.

$$\text{Curved Surface Area} = 2\pi r^2$$

$$\text{Total Surface Area} = 3\pi r^2$$

$$\text{Volume} = 2/3\ \pi r^3$$

The total surface area includes the circular base. The curved surface area does not include the base.

Example 10

Find the volume of a hemisphere of radius 6 cm in terms of π.

Volume = $2/3\ \pi r^3$.

= 2/3 × π × 6^3.

= 144π cm^3.

Answer: 144π cm^3

10. Prism

A prism is a solid with two identical and parallel bases. Its volume is obtained by multiplying the area of the base by the perpendicular height.

$$\text{Volume of prism} = \text{Area of base} \times \text{Height}$$

The exact surface-area formula depends on the shape of the base.

11. General Idea of Volume

For many solids, volume can be understood as the amount of three-dimensional space occupied by the object.

Volume = Base Area × Perpendicular Height

This directly applies to a prism and cylinder. A cone has one-third of the corresponding cylinder volume when both have the same base radius and height.

12. Relationship Between Cone and Cylinder

A cone and cylinder having the same base radius and height have a simple volume relationship.

Volume of cone = 1/3 × Volume of cylinder

Therefore, three cones of the same base and height have the same volume as one cylinder.

Example 11

A cylinder and cone have the same radius and height. If the cylinder volume is 900 cm^3, find the cone volume.

Cone volume = 1/3 × cylinder volume.

= 900/3.

= 300 cm^3.

Answer: 300 cm^3

13. Relationship Between Sphere and Hemisphere

A hemisphere is half of a sphere, so its volume is half the volume of a sphere of the same radius.

Hemisphere volume = 1/2 × sphere volume

14. Open and Closed Solids

In some questions, a container may be open at one end or at both ends. In such cases, do not automatically use the complete surface-area formula.

For example, an open cylindrical tank does not have the area of the missing circular base as part of its outer surface.

Always identify which surfaces are actually present.

15. Hollow and Combined Solids

Some questions involve a solid from which another solid has been removed. In such cases, calculate the required outer and inner measurements separately.

Remaining volume = Original volume − Removed volume

For combined solids, add the volumes of the parts when they are joined without overlap.

16. Conversion of Units

Volume conversions are especially important because a unit conversion is cubed.

1 m = 100 cm

1 m^3 = 1,000,000 cm^3

$$1 \text{ L} = 1000 \text{ cm}^3$$

$$1 \text{ mL} = 1 \text{ cm}^3$$

Do not use 100 cm^3 for 1 m^3. Since volume has three dimensions, the conversion factor is cubed.

Example 12

Convert 2.5 m^3 into cm^3.

1 m^3 = 1,000,000 cm^3.

2.5 m^3 = 2.5 × 1,000,000.

= 2,500,000 cm^3.

Answer: 2,500,000 cm^3

17. Capacity

Capacity is the amount a container can hold. It is closely related to volume.

$$1 \text{ litre} = 1000 \text{ cm}^3$$

$$1 \text{ kilolitre} = 1 \text{ m}^3$$

$$1 \text{ millilitre} = 1 \text{ cm}^3$$

Example 13

A tank has a volume of 2 m^3. What is its capacity in litres?

1 m^3 = 1000 litres.

2 m^3 = 2000 litres.

Answer: 2000 litres

18. Surface Area of a Cuboid — Detailed Example

Example 14

A closed rectangular box has length 20 cm, breadth 12 cm and height 8 cm. Find its total surface area.

TSA = 2(lb + bh + hl).

= 2(20×12 + 12×8 + 8×20).

= 2(240 + 96 + 160).

= 2 × 496 = 992 cm^2.

Answer: 992 cm^2

19. Finding a Missing Dimension

If volume and two dimensions are known, the missing dimension can be found by rearranging the volume formula.

Example 15

A cuboid has volume 720 cm^3, length 12 cm and breadth 10 cm. Find its height.

$V = lbh$.

$720 = 12 \times 10 \times h$.

$720 = 120h$.

$h = 6$ cm.

Answer: 6 cm

20. Cylinder — Finding Height

Example 16

A cylinder has radius 7 cm and volume 1540 cm^3. Find its height. Take $\pi = 22/7$.

$V = \pi r^2 h$.

$1540 = 22/7 \times 49 \times h$.

$1540 = 154h$.

$h = 10$ cm.

Answer: 10 cm

21. Cone — Finding Height

Example 17

A cone has radius 7 cm and slant height 25 cm. Find its height.

$l^2 = r^2 + h^2$.

$25^2 = 7^2 + h^2$.

$625 = 49 + h^2$.

$h^2 = 576$.

$h = 24$ cm.

Answer: 24 cm

22. Sphere — Finding Radius

Example 18

The surface area of a sphere is 616 cm^2. Find its radius. Take $\pi = 22/7$.

$4\pi r^2 = 616$.

$4 \times 22/7 \times r^2 = 616$.

$r^2 = 49$.

$r = 7$ cm.

Answer: 7 cm

23. Hemisphere — Surface Area

Example 19

Find the total surface area of a hemisphere of radius 7 cm. Take $\pi = 22/7$.

TSA = $3\pi r^2$.

= $3 \times 22/7 \times 49$.

= 462 cm^2.

Answer: 462 cm^2

24. Recasting and Melting Problems

In many mensuration questions, a solid is melted and recast into another shape. If there is no loss of material, the volume remains unchanged.

Volume before melting = Volume after recasting

Surface area generally changes after recasting, but volume remains constant.

Example 20

A metal cube of side 6 cm is melted and recast into a cuboid of length 9 cm and breadth 4 cm. Find its height.

Volume of cube = 6^3 = 216 cm^3.

Volume of cuboid = $9 \times 4 \times h = 36h$.

Since volume is unchanged, $36h = 216$.

$h = 6$ cm.

Answer: 6 cm

25. Comparing Volumes

When two solids have the same dimensions except for one variable, the ratio of their volumes can often be found without calculating each volume completely.

Example 21

Two spheres have radii in the ratio 2:3. Find the ratio of their volumes.

Volume of a sphere is proportional to r^3.

Therefore volume ratio = $2^3 : 3^3$.

= 8 : 27.

Answer: 8 : 27

26. Comparing Surface Areas

Example 22

Two spheres have radii in the ratio 2:3. Find the ratio of their surface areas.

Surface area is proportional to r^2.

Therefore area ratio = $2^2 : 3^2$.

= 4 : 9.

Answer: 4 : 9

27. Important Formula Table

Solid	Surface Area	Volume	Key Relation
Cuboid	TSA = 2(lbh+bh+hl)	lbh	Diagonal = $\sqrt{l^2+b^2+h^2}$
Cube	TSA = $6a^2$	a^3	Diagonal = $a\sqrt{3}$
Cylinder	TSA = $2\pi r(h+r)$	$\pi r^2 h$	CSA = $2\pi rh$
Cone	TSA = $\pi r(l+r)$	$1/3\pi r^2 h$	$l^2=r^2+h^2$
Sphere	$4\pi r^2$	$4/3\pi r^3$	
Hemisphere	$3\pi r^2$ (TSA)	$2/3\pi r^3$	CSA = $2\pi r^2$
Prism	Depends on base	Base area × height	

28. Common Mistakes

- Do not use square units for volume. Volume must be in cubic units.
- Do not forget that the total surface area of a cylinder includes both circular bases.
- For a hemisphere, curved surface area and total surface area are different.
- Do not use the cone's slant height in the volume formula; volume uses perpendicular height.
- When converting cubic units, cube the conversion factor.
- In an open container, do not include a surface that is not actually present.
- In melting and recasting questions, equate volumes, not surface areas.
- Check whether the question asks for curved, lateral, total surface area or volume.

29. Quick Revision Table

Concept	Formula / Key Point
Cuboid volume	lbh
Cube volume	a^3
Cylinder volume	$\pi r^2 h$
Cone volume	$1/3\pi r^2 h$
Sphere volume	$4/3\pi r^3$
Hemisphere volume	$2/3\pi r^3$
Cylinder CSA	$2\pi rh$
Cylinder TSA	$2\pi r(h+r)$
Cone CSA	πrl
Cone TSA	$\pi r(l+r)$
Sphere surface area	$4\pi r^2$
Hemisphere TSA	$3\pi r^2$
1 litre	1000 cm^3
1 m^3	1,000 litres
Recasting	Volume remains unchanged if there is no material loss

30. Final Formula Sheet

Cuboid: V = lbh

Cuboid: TSA = 2(lb + bh + hl)

Cube: $V = a^3$

Cube: $TSA = 6a^2$

Cylinder: $V = \pi r^2 h$

Cylinder: $CSA = 2\pi rh$

Cylinder: $TSA = 2\pi r(h+r)$

Cone: $V = 1/3\pi r^2 h$

Cone: $CSA = \pi rl$

Cone: $TSA = \pi r(l+r)$

Cone: $l = \sqrt{(r^2+h^2)}$

Sphere: $V = 4/3\pi r^3$

Sphere: $SA = 4\pi r^2$

Hemisphere: $V = 2/3\pi r^3$

Hemisphere: $CSA = 2\pi r^2$

Hemisphere: $TSA = 3\pi r^2$

Prism: V = Base area × Height

1 L = 1000 cm^3 = 1 dm^3

1 m^3 = 1000 L

31. Final Revision

For three-dimensional mensuration, first identify the solid and then decide whether the question asks for volume, curved or lateral surface area, or total surface area. Keep the dimensions in the same unit and use cubic units for volume.

The most useful habit is to draw a small sketch when a solid is described in words. Mark the radius, height, length, breadth and slant height before applying a formula.