Chapter 7: Statistics & Data Interpretation

Chapter 3 — Central Tendency | Detailed Notes & Solved Examples

1. Meaning of Central Tendency

A data set may contain many observations. Central tendency gives us one value that represents the general or central position of the data.

The three most important measures of central tendency are Mean, Median and Mode.

Central Tendency → Mean + Median + Mode

2. Why Do We Need Central Tendency?

- To summarise a large collection of observations with one representative value.
- To compare two or more groups of data.
- To understand the typical or central value of a distribution.
- To support decision-making and further statistical analysis.

3. Arithmetic Mean

The arithmetic mean, commonly called the average, is obtained by dividing the sum of all observations by the number of observations.

Mean = Sum of observations / Number of observations

$$\bar{x} = \Sigma x / n$$

Example 1

Find the mean of 8, 12, 15, 10 and 5.

Sum = 8 + 12 + 15 + 10 + 5 = 50.

Number of observations = 5.

Mean = 50/5 = 10.

Answer: 10

4. Mean When a Value Is Repeated

If the same value occurs many times, it can be written using frequency.

$$\text{Mean} = \Sigma fx / \Sigma f$$

Here x is the observation and f is its frequency.

Example 2

Find the mean of values 10, 20 and 30 having frequencies 2, 3 and 5 respectively.

$\Sigma f = 2 + 3 + 5 = 10$.

$\Sigma fx = 10\times2 + 20\times3 + 30\times5 = 20 + 60 + 150 = 230$.

Mean = 230/10 = 23.

Answer: 23

5. Weighted Mean

Sometimes different observations have different levels of importance. In such cases, a weighted mean is used.

Weighted Mean = $\Sigma wx / \Sigma w$

Here w represents the weight assigned to each value.

Example 3

A student scores 60 with weight 2 and 80 with weight 3. Find the weighted mean.

Weighted total = 60×2 + 80×3 = 120 + 240 = 360.

Total weight = 2 + 3 = 5.

Weighted mean = 360/5 = 72.

Answer: 72

6. Median

The median is the middle value of a data set after the observations have been arranged in ascending or descending order.

The first step is always to arrange the data. Without arranging the observations, the middle value cannot be identified correctly.

7. Median for an Odd Number of Observations

If the number of observations is n and n is odd:

Median = Value of the (n + 1)/2-th observation

Example 4

Find the median of 7, 2, 9, 5 and 4.

Arrange the data: 2, 4, 5, 7, 9.

There are 5 observations.

Middle position = (5+1)/2 = 3rd observation.

Median = 5.

Answer: 5

8. Median for an Even Number of Observations

If the number of observations is even, there are two middle values. The median is their arithmetic mean.

Median = Average of the n/2-th and (n/2 + 1)-th observations

Example 5

Find the median of 4, 9, 2, 7, 6 and 10.

Arrange: 2, 4, 6, 7, 9, 10.

There are 6 observations.

The two middle values are 6 and 7.

Median = (6+7)/2 = 6.5.

Answer: 6.5

9. Mode

The mode is the observation that occurs most frequently in a data set.

A data set can have one mode, more than one mode, or no mode if all values occur equally often.

Example 6

Find the mode of 4, 7, 5, 4, 8, 4, 6, 7.

4 occurs three times.

7 occurs two times.

The remaining values occur once.

Therefore, the mode is 4.

Answer: 4

10. Bimodal and Multimodal Data

If two values have the same highest frequency, the distribution is bimodal. If more than two values share the highest frequency, it may be called multimodal.

Example 7

Find the modes of 2, 3, 3, 4, 5, 5, 6.

3 occurs twice and 5 occurs twice.

All other values occur once.

Therefore the data is bimodal with modes 3 and 5.

Answer: 3 and 5

11. Relationship Between Mean, Median and Mode

For a moderately skewed distribution, an important empirical relationship is:

$$\text{Mode} \approx 3\ \text{Median} - 2\ \text{Mean}$$

This relationship is an approximation, not an exact identity for every data set.

$$\text{Mean} - \text{Mode} \approx 3(\text{Mean} - \text{Median})$$

Example 8

If the mean is 20 and the median is 18, estimate the mode using Mode ≈ 3 Median − 2 Mean.

Mode ≈ 3×18 − 2×20.

= 54 − 40.

= 14.

Answer: Approximately 14

12. Mean from a Frequency Table

x	f	fx	Cumulative f
10	2	20	2
20	3	60	5
30	5	150	10

From this table, $\Sigma f = 10$ and $\Sigma fx = 230$, so the mean is 23.

13. Median from a Discrete Frequency Distribution

For a discrete frequency distribution, first calculate cumulative frequency. Then locate the observation corresponding to the middle position.

For an odd total frequency N, locate the (N+1)/2-th observation. For an even total frequency, locate the middle positions carefully using the cumulative frequency.

Example 9

Values are 10, 20, 30, 40 with frequencies 2, 3, 4 and 1. Find the median.

Total frequency N = 2+3+4+1 = 10.

The 5th and 6th observations are needed.

Cumulative frequencies are 2, 5, 9, 10.

Both the 5th and 6th observations lie at value 30.

Median = 30.

Answer: 30

14. Mode from a Frequency Table

In a discrete frequency table, the mode is the value having the highest frequency.

Example 10

Values 5, 10, 15 and 20 have frequencies 3, 8, 5 and 2. Find the mode.

The highest frequency is 8.

It corresponds to value 10.

Answer: 10

15. Mean of Grouped Data

When observations are grouped into class intervals, the class mark is used as the representative value of each class.

$$\text{Class Mark } x_i = (\text{Lower Class Limit} + \text{Upper Class Limit})/2$$

$$\text{Mean} = \Sigma f_i x_i / \Sigma f_i$$

Example 11

Find the mean for classes 0–10, 10–20 and 20–30 with frequencies 2, 3 and 5.

Class marks are 5, 15 and 25.

$\Sigma f = 2+3+5 = 10$.

$\Sigma fx = 5\times2 + 15\times3 + 25\times5 = 10+45+125 = 180$.

Mean = 180/10 = 18.

Answer: 18

16. Assumed Mean Method

When the numbers involved are large, the assumed mean method can make calculation shorter.

$$\text{Mean} = A + \Sigma fd / \Sigma f$$

Here A is the assumed mean and $d = x - A$.

Example 12

For values 20, 30 and 40 with frequencies 2, 3 and 5, take assumed mean A = 30. Find the mean.

d values are −10, 0 and 10.

$\Sigma fd = 2(-10) + 3(0) + 5(10) = 30$.

$\Sigma f = 10$.

Mean = 30 + 30/10 = 33.

Answer: 33

17. Step-Deviation Method

When class intervals have equal width, the step-deviation method can simplify grouped-data calculations.

$$u = (x - A)/h$$

$$\text{Mean} = A + h(\Sigma fu / \Sigma f)$$

Here h is the common class width.

Example 13

For class marks 10, 20 and 30 with frequencies 2, 3 and 5, take A = 20 and h = 10. Find the mean.

u values are −1, 0 and 1.

$\Sigma fu = 2(-1) + 3(0) + 5(1) = 3$.

$\Sigma f = 10$.

Mean = 20 + 10(3/10) = 23.

Answer: 23

18. Effect of Adding a Constant

If the same constant is added to every observation, the mean, median and mode each increase by that constant.

Example 14

The mean of five observations is 18. If 7 is added to every observation, what is the new mean?

Every observation increases by 7.

Therefore the mean also increases by 7.

New mean = 18 + 7 = 25.

Answer: 25

19. Effect of Multiplying Every Observation

If every observation is multiplied by a constant k, the mean, median and mode are also multiplied by k.

Example 15

The median of a data set is 12. If every observation is multiplied by 3, find the new median.

Median changes in the same proportion.

New median = 12 × 3 = 36.

Answer: 36

20. Combined Mean of Two Groups

If two groups have different sizes, their combined mean is not obtained by simply averaging the two means. Each mean must be weighted by the number of observations.

$$\text{Combined Mean} = (n_1\bar{x}_1 + n_2\bar{x}_2) / (n_1 + n_2)$$

Example 16

The mean of 20 students is 60 and the mean of 30 students is 70. Find the combined mean.

Total of first group = 20×60 = 1200.

Total of second group = 30×70 = 2100.

Combined total = 3300.

Total students = 50.

Combined mean = 3300/50 = 66.

Answer: 66

21. Missing Observation from Mean

Example 17

The mean of 5 numbers is 18. Four of the numbers are 12, 16, 20 and 25. Find the fifth number.

Total of five numbers = 5×18 = 90.

Known total = 12+16+20+25 = 73.

Missing value = 90−73 = 17.

Answer: 17

22. Missing Frequency from Mean

Example 18

Values 10, 20 and 30 have frequencies 2, x and 3. If the mean is 22, find x.

Mean = Σfx/Σf.

(20 + 20x + 90)/(5+x) = 22.

110 + 20x = 110 + 22x.

2x = 0, so x = 0.

Answer: 0

23. Median and Outliers

The median is less affected by extremely large or small values than the mean. Therefore, the median can be useful when the data contains unusual observations.

Example 19

Which measure is generally less affected if one very large value is added to an otherwise balanced data set: mean or median?

A very large value can pull the mean upward.

The median depends mainly on the middle position.

Therefore the median is generally less affected.

Answer: Median

24. Choosing Mean, Median or Mode

Measure	When It Is Useful
Mean	When all observations should contribute and the data is reasonably balanced
Median	When the data is skewed or contains extreme observations
Mode	When the most common value or category is important

25. Mean, Median and Mode — Comparison

Feature	Mean	Median	Mode
Definition	Arithmetic average	Middle value	Most frequent value
Requires ordering?	No	Yes	No
Affected by extreme values?	Yes	Much less	Usually no
Can be more than one?	No	No	Yes
Works with frequencies?	Yes	Yes	Yes

26. Detailed Example — All Three Measures

Example 20

Find the mean, median and mode of 2, 4, 4, 5, 7, 8, 4, 10.

Arrange the data: 2, 4, 4, 4, 5, 7, 8, 10.

Mean = (2+4+4+5+7+8+4+10)/8 = 44/8 = 5.5.

There are 8 observations, so median = (4th + 5th)/2 = (4+5)/2 = 4.5.

The most frequent value is 4, so mode = 4.

Answer: Mean = 5.5; Median = 4.5; Mode = 4

27. Finding a Value When Mean Changes

Example 21

The mean of 6 observations is 15. If one observation of 10 is replaced by 22, find the new mean.

Original total = 6×15 = 90.

Increase in total = 22−10 = 12.

New total = 90+12 = 102.

New mean = 102/6 = 17.

Answer: 17

28. Mean After Removing an Observation

Example 22

The mean of 8 observations is 20. One observation, 13, is removed. Find the new mean.

Original total = 8×20 = 160.

New total = 160−13 = 147.

New number of observations = 7.

New mean = 147/7 = 21.

Answer: 21

29. Important Points to Remember

• Always arrange data before finding the median.

• Mean uses every observation, while median depends on position.

• Mode is the value occurring most frequently.

• For grouped data, use class marks when calculating the mean.

• Check the total frequency before calculating a mean or median.

• A combined mean must account for group sizes.

• Use the original value as the denominator for percentage change, not the new value.

30. Common Mistakes

• Taking the middle number before arranging the observations.

• Dividing the sum by the wrong number of observations.

• Forgetting to multiply each value by its frequency.

• Using class limits instead of class marks for grouped-data mean.

• Taking a simple average of two group means when group sizes are unequal.

- Assuming the mode must always be unique.
- Treating the empirical mean-median-mode relation as an exact rule for every data set.

31. Quick Revision Table

Concept	Formula / Key Point
Mean	$\Sigma x / n$
Frequency mean	$\Sigma fx / \Sigma f$
Weighted mean	$\Sigma wx / \Sigma w$
Odd median	$(n+1)/2$-th observation
Even median	Average of $n/2$-th and $(n/2+1)$-th observations
Mode	Most frequent value
Class mark	(Lower + Upper)/2
Combined mean	$(n_1\bar{x}_1+n_2\bar{x}_2)/(n_1+n_2)$
Empirical relation	Mode ≈ 3 Median − 2 Mean

32. Final Formula Sheet

$$\text{Mean} = \Sigma x / n$$

$$\text{Mean from frequency table} = \Sigma fx / \Sigma f$$

$$\text{Weighted Mean} = \Sigma wx / \Sigma w$$

$$\text{Odd Median} = (n+1)/2\text{-th observation}$$

$$\text{Even Median} = \text{Average of } n/2\text{-th and } (n/2+1)\text{-th observations}$$

$$\text{Class Mark} = (\text{Lower Limit} + \text{Upper Limit})/2$$

$$\text{Mean of grouped data} = \Sigma f_i x_i / \Sigma f_i$$

$$\text{Mean} = A + \Sigma fd/\Sigma f$$

$$\text{Mean} = A + h(\Sigma fu/\Sigma f)$$

$$\text{Combined Mean} = (n_1\bar{x}_1+n_2\bar{x}_2)/(n_1+n_2)$$

$$\text{Mode} \approx 3\ \text{Median} - 2\ \text{Mean}$$

33. Final Revision

Central tendency gives a single value that represents the centre of a data set. Mean is the arithmetic average, median is the middle value after arranging the data, and mode is the most frequently occurring value.

For calculation questions, first identify the type of data, then choose the appropriate formula. For frequency tables, always check the total frequency and for grouped data remember to use class marks.