SET THEORY

Introduction to Sets

A set is a well-defined collection of distinct and unique elements or members. It is denoted using capital letters like A, B, C, and the elements are written within curly braces `{}`.

- If an element 'x' belongs to a set A, it is written as **$x \in A$**.
- If 'x' does not belong to A, it is written as **$x \notin A$**.

Examples:

- A = {1, 2, 3, 4} is a set of natural numbers less than 5.
- B = {a, e, i, o, u} is the set of vowels in English.

Important Notes:

- Elements are not repeated.
- Order does not matter: {1,2,3} = {3,2,1}.
- A set can be **finite** or **infinite**.
- A set can be represented either in **roster form** or **set-builder form**.

Sets are the foundation of modern mathematics, especially in logic, algebra, and data organization. Understanding them is crucial for higher studies in mathematics.

Methods of Representing Sets

Roster or Tabular Form

The elements of a set are listed explicitly, separated by commas, and enclosed within curly braces.

Example: A = {2, 4, 6, 8}

Set-Builder Form

The set is defined by a property that its members must satisfy.

Example: B = {x | x is a prime number less than 10} = {2, 3, 5, 7}

Types of Sets (With Examples and Figures)

Type	Definition	Example
Empty Set	A set with no elements	∅ or { }
Singleton Set	A set with exactly one element	{5}
Finite Set	A set with countable (finite) elements	{1, 2, 3}
Infinite Set	A set with an uncountable number of elements	{x,x ∈ N}
Equal Sets	Two sets with identical elements	{1,2,3} and {3,2,1}
Subset	A ⊆ B: every element of A is also in B	{1,2} ⊆ {1,2,3}
Proper Subset	A ⊂ B and A ≠ B	{1,2} ⊂ {1,2,3}
Universal Set	The set containing all elements under discussion	U = {1,2,3,4,5}
Power Set	The set of all subsets of a given set	P({a,b}) = {∅,{a},{b},{a,b}}

More Examples and Illustrations:

- If A = {1, 2}, then P(A) = {∅, {1}, {2}, {1,2}}
- If U = {1, 2, 3, 4, 5}, and A = {2,3}, then A' = {1,4,5}
- For every set A: A ⊆ A, and ∅ ⊆ A

Important Formula:

- Number of subsets of a set with n elements = 2^n

Example: A = {1,2,3} → 2^3 = 8 subsets

Formula 2: Number of Proper Subsets = 2^n - 1

We subtract 1 because we exclude the set itself.

Example: A = {1, 2} → Proper subsets = {}, {1}, {2} → Total = 3 = 2^2 - 1

Formula 3: Subsets of size r = C(n, r)

This is the number of ways to choose r elements from n. Also written as nCr.

Example: A = {a, b, c}, number of subsets of size 2 = C(3, 2) = 3

Those subsets are: {a, b}, {a, c}, {b, c}

Formula 4: Total Subsets using Summation = Σ C(n, r) from r=0 to n

This means we add all the combinations of all sizes.

Example: A = {1, 2, 3} → C(3,0) + C(3,1) + C(3,2) + C(3,3) = 1 + 3 + 3 + 1 = 8

Formula 5: Number of Non-empty Subsets = 2^n - 1

We remove the empty set from all subsets.

Example: A = {a, b} → Total = 4 → Non-empty = 4 - 1 = 3

Formula 6: Power Set Cardinality = |P(A)| = 2^n

This is just the count of all subsets in the power set.

Example: A = {x, y} → Power set P(A) = {}, {x}, {y}, {x, y} → Total = 4

Venn Diagrams (Explanation with Figures)

Venn diagrams are diagrams that show relationships between different sets using overlapping circles. These are used for visualizing problems and solving them easily.

Universal Set (U): represented by a rectangle. **Each set:** represented by a circle inside U.

Union (A ∪ B):

All elements in A or B or both.

Intersection (A ∩ B):

Common elements between A and B.

Difference (A - B):

Elements in A but not in B.

Complement (A'):

Elements not in A with respect to U.

Figures usually use shaded regions to show these relationships. In NDA, questions using Venn diagrams often ask for counts using union/intersection formulas.

Set Operations (Formulas and Examples)

Union:

A ∪ B = {x | x ∈ A or x ∈ B} **Example:** A={1,2}, B={2,3} ⇒ A∪B = {1,2,3}

Intersection:

A ∩ B = {x | x ∈ A and x ∈ B} **Example:** A={1,2,3}, B={2,3,4} ⇒ A∩B = {2,3}

Difference:

A - B = {x | x ∈ A and x ∉ B} **Example:** A={1,2,3}, B={2,3} ⇒ A−B = {1}

Complement:

A' = U - A = {x | x ∈ U and x ∉ A} **Example:** U={1,2,3,4}, A={2,3} ⇒ A'={1,4}

Cardinal Number Formula:

- n(A ∪ B) = n(A) + n(B) – n(A ∩ B)
- n(U) = n(A) + n(A')
- n(A - B) = n(A) - n(A ∩ B)

Properties of Sets (Expanded with Examples)

Commutative:

- A ∪ B = B ∪ A
- A ∩ B = B ∩ A

Associative:

- (A ∪ B) ∪ C = A ∪ (B ∪ C)
- (A ∩ B) ∩ C = A ∩ (B ∩ C)

Distributive:

- A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)
- A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

Identity:

- A ∪ ∅ = A
- A ∩ U = A

Complement:

- A ∪ A' = U
- A ∩ A' = ∅

These properties are useful for simplifying set expressions and solving equations.

De Morgan's Laws (with Visual and Tabular Examples)

Laws:

1. (A ∪ B)' = A' ∩ B'
2. (A ∩ B)' = A' ∪ B'

Example: U = {1,2,3,4,5,6}, A = {1,2,3}, B = {3,4,5}

- A ∪ B = {1,2,3,4,5} → (A ∪ B)' = {6}
- A' = {4,5,6}, B' = {1,2,6} → A' ∩ B' = {6} Verified

Power Set

- P(A) = Set of all subsets of A
- If A has n elements, then |P(A)| = 2^n

Example: A = {x,y} ⇒ P(A) = {∅, {x}, {y}, {x,y}}

Cartesian Product

- A × B = {(a,b) | a ∈ A, b ∈ B}
- |A × B| = |A| × |B|

Example: A = {1,2}, B = {x,y} ⇒ A × B = {(1,x), (1,y), (2,x), (2,y)}

Cartesian products are used in defining relations and functions.

Solved Examples (Expanded)

Example 1: If A = {1,2,3}, B = {2,3,4}, find A ∪ B and A ∩ B. ⇒ A ∪ B = {1,2,3,4}, A ∩ B = {2,3}

Example 2: If A = {x | x is a multiple of 2 less than 10}, write A. ⇒ A = {2,4,6,8}

Example 3: In a class of 50 students, 30 like English, 25 like Math, 10 like both. Find number who like at least one subject. ⇒ n(E ∪ M) = 30 + 25 – 10 = 45

Example 4: Prove that (A ∩ B)' = A' ∪ B' U = {1,2,3,4,5}, A = {1,2}, B = {2,3} A ∩ B = {2} ⇒ (A ∩ B)' = {1,3,4,5} A' = {3,4,5}, B' = {1,4,5} ⇒ A' ∪ B' = {1,3,4,5}

Practice Questions

1. A = {1,2,3}, B = {2,3,4}. Find A – B and B – A.
2. List all subsets of {a,b,c}.
3. Draw Venn diagram showing $A \subset B \subset U$.
4. Prove De Morgan's Laws with example.
5. In a group of 100 people, 60 watch cricket, 45 watch football, and 25 watch both. Find how many watch at least one.
6. If A = {x | x is a prime number ≤ 10}, list all subsets.
7. Write the power set of {1,2} and verify the number of subsets.
8. If n(A) = 25, n(B) = 20, and $n(A \cap B) = 10$, find $n(A \cup B)$.
9. Prove or disprove: $(A \cup B) \cap A = A$
10. Give an example of a singleton set and a power set of that singleton.

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